code is not complete, it communicates the essential ideas of the discretization techniques and solvers, and is a good starting place from which to implement the methods under consideration.

The target audience of the book is postgraduates and experienced researchers. While the book reviews the background material to facilitate the discussion and mathematical results, some knowledge in functional analysis, numerical PDEs, mathematical modeling, and, in some parts, differential geometry would benefit the reader. The writing style of the book is terse. While I am a fan of this style, I think that additional details in the background material and proofs would aid the presentation. Each page is quite dense with information, and I often found myself rereading certain passages with pen and paper. It is not a book to bring to the beach. Besides this minor quibble, I think this is an excellent monograph describing methods found at the intersection of numerical PDEs and the calculus of variations.

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Active Subspaces: Emerging Ideas for Dimension Reduction in Parameter Studies. *By Paul G. Constantine.* SIAM, Philadelphia, 2015. $39.00. x+100 pp., softcover. ISBN 978-1-611973-85-3.

This book is a very nice contribution to a special topic in model dimension reduction, a discipline that has grown fast in the last decade and includes computational reduction strategies, efficient geometrical parametrization, real-time computing and visualization, and last, but not least, the topic of the reviewed text: dimension reduction in parameter studies. For various reasons this is the preliminary step taken when facing a complex parametrized problem.

The text has been carefully crafted in order to arouse interest and to interact with readers in a pedagogic sense, so that by the end of the book they are already trying to apply the proposed techniques to their problems of interest. The style is enthusiastic and fresh, and the text should be useful and interesting for computational scientists and interdisciplinary researchers.

Parameter studies are growing in popularity in computational science due to the increasing complexity of engineering problems which demand efficient many query input-output relationships for optimization, uncertainty quantification, and sensitivity analysis problems. The number of parameters involved is becoming larger and larger and must be reduced, which is often done by limiting the study to the most important parameters. Dimension reduction must be practical and well motivated, easy to implement, and easy to interpret, in order to have an impact on real applications in engineering and applied sciences. Dimension reduction techniques in parameter studies should be not heuristic but theoretically well founded with rigorous performance guarantees, and this should hold not only for toy problems.

Active subspaces, as described in the text, offer a more general approach to reducing the parametric study's dimension by identifying sets of important ("magic") directions in the space of all the inputs.

In the first chapter easy-to-implement procedures to discover whether a given model admits an active subspace are presented. The chapter gives a quick introduction to active subspaces for engineers involved in high-dimensional parameter studies. Preliminary ingredients are introduced as well as some historical stories recalling regression models in statistics. Three algorithms are presented: active subspace estimation with gradients, with local linear models, and with a global linear model. The quantities computed by the proposed algorithms are used to decide whether the model is eligible for dimension reduction. A gap between eigenvalues is used to understand the separation between active and inactive subspaces through several examples.

The second chapter focuses on active subspaces from a wider perspective dealing with algorithms and parametrized simulations for uncertainty quantification. This part of the text provides a strong motivation for dimension reduction in parameter spaces in order to combine assimilated data with increasingly complex physical and mathematical models, as well as rich, coupled, and interconnected systems. This chapter also makes a connection with computational model reduction to face the curse of dimensionality.

The third and fourth chapters are dedicated to the technical details of defining, developing, and computing effective strategies for active subspaces. Some effective examples, algorithms, and recipes are provided, and a few approximations and variants provide deeper perspective. In particular, Chapter 4 is dedicated to the exploitation of active subspaces with the aims of guaranteeing efficiency and, at the same time, merging new ideas. Also some ideas in progress are widely illustrated.

The fifth chapter shows some interesting applications of active subspaces in action: the hypersonic scramjet parametric study, a photovoltaic solar cell, and a more classical airfoil shape optimization. All are characterized by complex input-output relationships and they help maintain the reader's interest in the real world problems and applications of this topic.

The last chapter, after some conclusions, introduces current research topics such as multiple outputs, coupled systems, e.g., multiphysics, and also, last but not least, a sort of possible dualism between high-order and low-order models, wondering whether the active subspaces found in a low fidelity setting can be transferred to the high fidelity one.

The website activesubspaces.org provides scripts, utilities, and updates on the topic and its developments.

The text is very interesting and represents a rapid approach to dimension reduction in parameter spaces for computational scientists and engineers working on complex parametric systems. This field will grow further in importance in the next few years.

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The Shapes of Things: A Practical Guide to Differential Geometry and the Shape Derivative. *By Shawn W. Walker*. SIAM, Philadelphia, 2015. $74.00. x+152 pp., softcover. ISBN 978-1-611973-95-2.

Shawn Walker's book is nicely produced and contains some beautiful and helpful color graphics. As a differential geometer, I was interested to read it and gain some appreciation for the applied nature of shape theory; to be honest, I had expected more interweaving of numerical analytic material, as that is the author's expertise. I found the book mildly interesting, but cumbersome and ponderous in many places and quite difficult to read and absorb. Moreover, I found some substantive mathematical errors, along with notation and sign conventions that would be unnecessarily confusing to anyone familiar with a bit of differential geometry. Indeed, my immediate reaction when I started reading the book was to inquire why no differential geometry expert had reviewed the book *before* publication.

In undergraduate differential geometry courses we study notions of curvature for curves and surfaces (predominantly in $\mathbb{R}^3$), which have local definitions and global consequences. The first half of Walker's book is devoted to a cursory (but in some places belabored) summary of the essential material from such a course. Unless one discusses first (and perhaps second) variation of arclength or surface area in the undergraduate geometry course, the curves and surfaces are static. However, if one wants to speak of the curve with least arclength (say, with fixed endpoints or enclosing a certain fixed area) or of the surface with least area (say, with a given boundary curve, like dipping a wire in soap solution to get a soap film, or enclosing a given volume, like blowing a soap bubble), then one might think of the curve or surface as moving, with constraints, and consider the calculus problem of determining whether we have a minimum for all such possible motions. This is the calculus of variations and is an instance of the shape derivative Walker treats in the latter half of his book. (See section 5.4 of doCarmo's *Differential Geometry of Curves and Surfaces* or Chapter 7 of Oprea's *Differential Geometry and its Applications*.)

First, Walker develops the intrinsic calculus of functions and vector fields on a surface in an extrinsic manner. For example, consider a surface $M \subset \mathbb{R}^3$ and a function f on