On Active Subspaces in Car Aerodynamics

Carsten Othmer
Electronics Research Laboratory, Volkswagen Group of America Inc., Belmont, CA 94002, USA

Trent Lukaczyk
Stanford University, Stanford, CA 94305, USA

Paul G. Constantine
Colorado School of Mines, Golden, CO 90211, USA

Juan J. Alonso
Stanford University, Stanford, CA 94305, USA

The Active Subspace Method (ASM) is an emerging set of tools for dimensionality reduction in complex physical systems. It allows to discover low-dimensional trends in the quantity of interest by exploiting redundancies in the input variables and combining them linearly into so-called active variables. The purpose of this study is to assess the applicability and the benefit of the ASM in car aerodynamics. To that end, we apply the ASM to drag and lift computations of three different parameterized vehicle geometries of increasing complexity. We thereby assess the impact of adjoint-based gradient inaccuracies on the results of the ASM, devise and validate a methodology to apply the ASM in the absence of adjoint-based gradients, and exemplify the practical use of this methodology in car aerodynamics. For all investigated cases, the ASM reveals that a large portion of the overall variability of drag or lift is captured already by an active subspace of dimension one, thus providing physical insight into the main shape parameter dependencies. By projection into an active subspace of a suitably chosen dimension larger than one, it is demonstrated that the predictive accuracy of surrogate models for drag and lift can consistently be improved.

I. Introduction

In view of its impact on the global climate, the automotive industry is undertaking major efforts to further reduce the CO₂ footprint of its vehicle fleets – both by electrification of the powertrain and via continuous efficiency increases of internal combustion (IC) engines. Next to vehicle weight and rolling resistance, the aerodynamic drag is the third major factor determining the range of electric vehicles and the fuel consumption of IC-powered cars. Aerodynamic shape optimization is therefore receiving enhanced attention in automotive R&D again.

Optimization of the external aerodynamics of entire vehicles is a peculiar task: Except for dedicated low emission cars, vehicle shapes are to a high degree driven by esthetic considerations rather than aerodynamic performance. Since esthetic requirements can impossibly be transformed into mathematical constraints, the automatic optimization of vehicle shapes, which is common practice in aerospace applications,¹ is not an option, unless restricted to small portions like spoilers, mirrors, or to the underbody.

Under these circumstances, the adjoint-based computation of surface sensitivity maps – without a gradient-driven automatic shape update – has been demonstrated² to fit very nicely into the specific requirements of the aerodynamic development of vehicle shapes: The information contained in these maps, i.e. the derivative of drag or lift with respect to local shape deformations, provides the designer with concrete suggestions for aerodynamic improvements, and it is in his/her hand to translate this information into an aerodynamically improved shape without compromising the esthetics.
While the sensitivity maps already constitute a strong link between the styling process and the aerodynamic design process, the further coupling of these two development disciplines is inhibited by their disparate time scales: The styling evolves on a daily basis, whereas the computation of a sensitivity map for a single vehicle design requires of the order of a full week (including model preparation, computation and post-processing). As a result, the aerodynamic simulation as of today cannot keep pace with the styling process. In order for the vehicle design to fully benefit from the valuable information contained in the sensitivity maps, an Interactive Aerodynamic Design (IAD) process, which allows to compute – at least approximate – sensitivity maps in near real-time, has to be developed.

On the pursuit of this long-term objective, Volkswagen Group Research in cooperation with the Dept. of Aeronautics and Astronautics at Stanford University are looking into the possibility of leveraging existing methods of Response Surface Modeling (especially Gaussian Process Regression – GPR) and Reduced Order Modeling (ROM) for real-time aerodynamic computations of parameterized vehicle geometries. Since both of these modeling techniques suffer from the “curse of dimensionality” in the sense that in the typically encountered high-dimensional design spaces of industrial shape optimization problems they incur huge amounts of computational cost for building a reliable model and often fail to meet the required predictive accuracy, we are focusing on dimension reduction techniques as a first building block of the intended IAD tool.

As demonstrated in recent studies, the Active Subspace Method (ASM) pioneered by Constantine et al., is a promising method for dimension reduction in the aerodynamic shape optimization context. It performs a Principal Component Analysis on the gradient of the function of interest, and thus determines the most important directions of the cost function variability as the principal components of this gradient. These directions are linear combinations of the original design variables and are referred to as active variables. By choosing an appropriately small number of active variables, a low-dimensional active subspace can be constructed that captures already the majority of the cost function variability. A well-approximated lower-dimensional model of the original problem is then obtained via projection of the original variables into the active subspace.

The current investigation focuses on the applicability and the benefit of the ASM for dimension reduction in vehicle shape optimization problems. The challenge of transferring the ASM from previous successful applications in the aerospace context to vehicles is two-fold and related to (1) gradient accuracy in the presence of separated flows, and (2) a qualitatively different way of geometry parameterization of car shapes vs. airfoils:

1. For the existing implementations of adjoint-based gradient computations for complex bluff bodies like cars, it has been shown that the gradients tend to be qualitatively correct, but are quantitatively not reliable – the main reason being the massive flow separation right in the region of highest drag impact. It is an open question to which degree these gradient inaccuracies impair the determination of the active subspace for bluff body geometries with separated flows.

2. In the case of adjoint-based shape optimization, geometric parameterizations for airfoils or entire aircraft bodies typically involve “localized” perturbations like Hicks-Henne bumps or a dense lattice of free-form deformation control points. The action of the ASM on this kind of parameterization consists essentially in combining these local perturbations into actual features of the airfoil like camber, angle of attack, or twist – hence the massive potential for dimensionality reduction via ASM in these cases. For car shapes, however, typical parameterizations are already feature-based and involve spatially extended deformations like the length of the trunk, the height of the roof, the angle of the wind screen etc. In other words, a dimension reduction takes place already in the actual set up of the design variables, and it is not clear if the ASM can provide any added-value beyond that, by combining these feature-based parameters into a yet smaller set of active variables.

After a concise introduction into the theory behind the ASM in the next section, we answer these questions by applying the ASM to parameterized bluff bodies of three different degrees of complexity: Small deformations of the Ahmed body with almost linear effects on drag are used to assess the impact of gradient inaccuracies onto the ASM results. For this case, we contrast the output of the ASM based on inaccurate gradients stemming from an adjoint method with those derived from a surrogate model fitted to the Ahmed body drag data. As the latter method of obtaining gradients turns out to be more reliable, at least in the case of bluff bodies computed with currently available adjoint implementations, we further validate and
scrutinize this procedure by applying it to a low-dimensional, localized parameterization of a realistic car geometry. The benefit of the ASM for real-life vehicle shape optimization is finally evaluated on the basis of a global parameterization of a production passenger car taken from the series development at Volkswagen.

II. The Active Subspace Method

Excellent overviews of the ASM have previously been provided by Constantine and co-workers. However, for the sake of self-containedness of this report, we concisely review the basic concept and the related nomenclature in the following.

At the heart of the ASM is the averaged outer product $C$ of the cost function gradient $\nabla f$, which is approximated by a sum over the $M$ computed samples as

$$ C \approx \frac{1}{M} \sum_{i=1}^{M} \nabla f(x_i) \nabla f(x_i)^T, $$

where $x \in \mathbb{R}^N$ is the vector of parameters forming the $N$-dimensional design space, and $i$ is the sample index. Given the eigendecomposition of the approximated $C$,

$$ C = W \Lambda W^T $$

the eigenvectors $w_j$ are sorted according to descending eigenvalues $\lambda_j$. The first $n$ eigenvectors are taken to construct $U$ and to partition $W$ as

$$ W = \begin{bmatrix} U & V \end{bmatrix}, $$

$U$ forms the basis of the $n$-dimensional active subspace. By projecting the original design variables $x$ into the active subspace $\text{span}(U)$, we obtain the active variables $y$:

$$ y = U^T x, \quad y \in \mathbb{R}^n. $$

The function of interest $f$ can now be formulated in the active subspace as

$$ f(x) \approx g(U^T x) = g(y). $$

The most appropriate choice of $n$ depends on the eigenvalue spectrum and should be based on the following observations:

1. The mean squared 2-norm of the gradient along the active variables equals the sum of the corresponding eigenvalues:

$$ \frac{1}{M} \sum_{i=1}^{M} \nabla g(y_i)^T \nabla g(y_i) = \lambda_1 + \cdots + \lambda_n. $$

2. The error due to incomplete sampling in determining an active subspace of dimension $n$ is inversely proportional to the spectral gap $\lambda_n - \lambda_{n+1}$.

For the determination of the active subspace dimension $n$, it is therefore common practice to look for gaps in the eigenvalue spectrum and transform the original design space accordingly.

III. Applications

In this section, we apply the ASM to three different test cases in order to assess the potential and the limitations of this method in the context of vehicle shape optimization: (A) the classical Ahmed body, (B) a realistic car geometry subjected to low-dimensional, local geometric perturbations, and (C) a global parameterization of a production passenger car.
Figure 1. Ahmed body test case: (a) Geometry parameters $p_0$ to $p_7$. (b) RANS-based flow solution for the baseline configuration (pressure field on the body, velocity in the symmetry plane and on the street).

III.A. Ahmed body

For this initial study, the 25° slant angle case of the Ahmed body is parameterized as depicted in Fig. 1a. Each of the four edges defining the rear face can move independently by up to 10mm in two orthogonal directions, giving a total of eight parameters $p_0$ to $p_7$. Intentionally, symmetry was not enforced, in order to check if the ASM results would reflect the similarity between $p_2$ and $p_6$, and between $p_3$ and $p_7$, respectively.

A hybrid mesh with a total cell count of roughly 700,000 elements was generated, including 30 prism layers with an average $y^+$ value of 1.1 and a refinement in the wake zone. Based on a Latin Hypercube sampling, 32 direct and adjoint Reynolds-Averaged Navier-Stokes (RANS) simulations were carried out with OpenFOAM® using the Spalart-Allmaras turbulence model (see Fig. 1b for the flow around the baseline geometry). As shown in Fig. 2a, the computed drag values range from 84 to 100%, with the latter being the baseline configuration corresponding to $p_i = 0$ for all $i$.

Figure 2. (a) Training samples for the Ahmed body test case. (b) Computed gradients for the baseline configuration.

The employed adjoint solver is based on the continuous adjoint approach and makes use of the “frozen turbulence” assumption. From previous experience with this solver being applied to bluff body cases with flow separation, it is well-known that the computed sensitivities are qualitatively correct (they have the right sign and the right relative scaling), but quantitatively, i.e. in terms of magnitude, they are usually far off. This behavior is confirmed by a finite difference check for the baseline configuration (Fig. 2b): The

American Institute of Aeronautics and Astronautics
relative importance of the individual parameters are reflected correctly in the adjoint-based sensitivities (both for the 1st and the 2nd order adjoint), but their magnitudes are off by at least a factor of two.

While this error can be reduced to a certain degree by differentiating the turbulence model, significant inaccuracies for separated flows persist, especially in the vicinity of sharp edges, and remain to be investigated.

The question we want to answer here is in how far the results of the ASM are impaired by the – for the time being – unavoidable adjoint-based gradient inaccuracies of bluff bodies exhibiting massive flow separation. To that end, we will contrast the Ahmed body ASM results based on the gradients from the adjoint computation with those derived from a surrogate model fitted to the Ahmed body drag data.

Following the procedure outlined in the previous section, we first employ the adjoint-based gradients to construct the gradient covariance matrix and compute its eigendecomposition. The magnitudes of the eigenvalues (Fig. 3a) exhibit a rather slow decay, with the second eigenvalue being 25 times smaller than the largest. According to the eigenvector that corresponds to the largest eigenvalue (see Fig. 3b), the overall drag behavior is dominated by parameter $p_4$, followed by parameters $p_2$ and $p_6$, the two of which have similar impacts – as they should because of symmetry (cf. Fig. 1a).

![Figure 3. (a) Eigenvalues of the covariance matrices for adjoint-based and GPR-model-based gradients, respectively. (b) Eigenvectors 1 and 2 when using adjoint-based gradients.](image)

Upon projecting the 8-dimensional data into the first active subspace (the one spanned by the first eigenvector), the 1D trend in the data is clearly visible (Fig. 4a), albeit with a sizeable spread. Fitting a 1D response surface based on Gaussian Process Regression (GPR) to the projected data yields a testing error of 1.3% (here by modeling the covariance as a zero mean Gaussian of the Euclidean distance between sample points, with a nugget term corresponding to independent identically-distributed Gaussian noise). Indeed, the slow decay of eigenvalues indicated already that the first active subspace does not suffice to cover the drag variability extensively. When projecting the data into the subspace spanned by the first two eigenvectors, the GPR model accuracy improves accordingly and the model error reduces to 0.8% (Fig. 4b). The extension to a 3-dimensional active subspace does, however, not result in a further improvement.

The extent of the shape perturbations was intentionally chosen to be small, such that the drag behavior is almost linear in the geometric parameters $p_i$. As a result, a very accurate 8-dimensional GPR model for drag as a function of $p_0$ to $p_7$ can be fitted to the data. A “leave-one-out” cross-validation of this model produces a mean testing error of less than 0.1%, and the gradient derived from this surrogate model for the baseline configuration matches the CFD finite difference results very well (Fig. 2b). Therefore, the model-derived gradients will now serve to compute the “true” active subspaces for a comparison with the adjoint-based results.

As can be seen from Fig. 3, there is almost a factor of 1000 between the largest and the second-largest eigenvalue of the model-based gradient covariance matrix, clearly indicating a dominant one-dimensional subspace – in contrast to the rather flat eigenvalue spectrum of the adjoint-based gradients. The first eigenvector confirms the dominating role of $p_4$ and the symmetric contributions of $p_2$ and $p_6$, as depicted in Fig. 5a. From this figure, we also infer that the adjoint-based gradients overestimate the role of $p_4$ as compared to $p_2$ and $p_6$. Interestingly, this overestimation is partly compensated by the second adjoint-
based eigenvector (Fig. 3b), where $p_2$ and $p_6$ on the one hand and $p_1$ on the other enter with opposite signs, which in turn explains the observed significant improvement in model accuracy when increasing the active subspace dimension from one to two (Fig. 4b). Accordingly, while the deviation between the first model-based eigenvector and its adjoint-based counterpart amounts to a sizeable $30.6^\circ$ (in the 8-dimensional $p_i$-space), it drops to $16.9^\circ$ between the first model-based eigenvector and the plane spanned by the first two adjoint-based eigenvectors.

The projection of the data into the first model-based active subspace (Fig. 5b) confirms the essentially one-dimensional behavior of the drag. A 1D GPR model fit to the projected data yields a testing error of less than 0.12%, as compared to about 1.3% for the adjoint-based 1D active subspace projection (Fig. 4a).

Given the confidence now gained in the model-based gradient information, we can use it to quantify the inaccuracy of the adjoint-based gradients and relate it to the inaccuracy of the ASM outcome. For the partial derivatives of the drag with respect to each of the parameters $p_i$, Fig. 6a shows for all 32 samples a comparison between those obtained from the adjoint method and those derived from the surrogate model.
With the Pearson correlation coefficients ranging from -0.44 to +0.37, these two sets of gradients hardly exhibit any correlation. However, recalling that the basis of the active subspace computation is the gradient covariance matrix, we infer that it is not the individual accuracy of each of the gradient components that determines the active subspace accuracy, but rather their relative scaling. Therefore, a more reasonable way of looking at gradient accuracy in terms of its impact on the determination of active subspaces is probably Fig. 6b: a histogram of the angles (in the 8-dimensional \( p_i \)-space) between the adjoint-based and the model-based gradient. Even when cast in this quantity, gradient accuracy is still quite poor. We note, however, that the average deviation is around 30\(^\circ\), which corresponds to the deviation of the first adjoint-based eigenvector from its model-based counterpart (see above). If this can be generalized as a rule of thumb for the impact of gradient inaccuracies onto active subspace computations remains, however, to be confirmed by rigorous error estimation methods, e.g. along the lines of Constantine and Gleich.\(^9\)

![Gradient Correlation](image1)

![Gradient Correlation: Angles](image2)

Figure 6. Gradient accuracy: (a) Correlation between adjoint-based and model-based gradient. The numbers are the corresponding Pearson correlation coefficients. (b) Distribution of angles between these two sets of gradients.

As a result from the application of the ASM to the Ahmed body case, we conclude for bluff body geometries with massive flow separation, that currently available adjoint-based gradients are too inaccurate to justify their usage for the determination of active subspaces: A multi-dimensional adjoint-based active subspace construction was outperformed by a simple linear regression. Given this poor gradient accuracy, it almost comes as a surprise that the adjoint-based ASM did recover some correct trends: The qualitative influences of \( p_2 \), \( p_4 \) and \( p_6 \) were captured as well as the similarity between \( p_2 \) and \( p_6 \). Moreover, the over/underestimation of the impact of these parameters in the first active subspace was partly compensated for in the 2nd. While these findings are promising, feeding the ASM with gradients derived from a surrogate model turned out to be the wiser strategy – at least in this instance. With the following more complex examples, we will further validate and scrutinize this strategy.

### III.B. Low-dimensional localized car shape parameterization

The geometry underlying this test case is the 6th generation of the Volkswagen Passat Limousine (Fig. 7). It is a full-fledged CFD model including a detailed underbody, but with a sealed engine bay to exclude underhood flow. What makes up this test case, is the combination of geometric complexity on the one hand with localized, controlled shape deformation on the other. It is simple enough to check the validity of the surrogate model-aided procedure proposed in the previous section yet realistic enough to assess the practical value of the ASM results for car geometries of industrial complexity.

Symmetric free-form deformation boxes were set up in ANSA\(^{\text{®}}\),\(^{14}\) with two control points defined at the end and in the center of the transition line between the roof and the rear window (Fig. 7): Point A can move in all three Cartesian directions, and the center point S in the \( x \) and \( z \) directions only, thus giving precise control over the shape of the rear roof/window area. Each of the five design variables \( A_x, A_y, A_z, S_x \) and \( S_z \) can be perturbed by ±20mm.

A Latin Hypercube of 89 training and 10 testing RANS simulations was computed using the Spalart-Allmaras turbulence model on a hybrid mesh of 17M cells and an average \( y^+ \) of 50. Despite the sizeable
Figure 7. Passat test case: (a) RANS-based flow solution for the baseline configuration (pressure field on the body, velocity in the center plane). (b) Free-form deformation boxes defined to control the rear roof/window area: The pink boxes are tied to the green boxes via symmetry constraints. Point A can be moved freely in 3 dimensions, point S only in the center plane (blue edges).

geometric perturbations (in terms of real car design), the drag spreads by only about 1%, the largest part of which is properly covered by the testing data set (see Fig. 8a).

As outlined above, we do not rely on adjoint-based gradients for this test case, but rather fit a surrogate model to the drag data and derive the gradients from that. Fig. 8b depicts the obtained high-quality 5-dimensional GPR model exhibiting a testing error of only 0.03%. The gradients extracted from this model are quite constant throughout the sampled parameter space (Fig. 9a). Apparently, all the designs are actually lying on a 5-dimensional hyperplane, and thus exhibit large potential for dimensionality reduction via the ASM. The spectrum of the eigenvalues (Fig. 9b), with a factor of almost 1000 between the biggest two, indicates indeed a dominant one-dimensional active subspace. The projection of the 5D data into this 1D active subspace results in a testing error of 0.03% (Fig. 10a), which is of the same size as the error of the full 5D model.

Figure 8. (a) Passat drag data samples. (b) 5D GPR model.

The one active variable that governs the overall drag behavior, i.e. the eigenvector corresponding to the largest eigenvalue, is shown in Fig. 10b, along with the perfectly agreeing linear regression coefficients obtained again for verification. It defines the “shape deformation mode” with the largest impact on drag. The visualization of this kind of modes directly on the car surface as in Fig. 11 delivers physical insight and is of great practical value. In this rather simple case, we can see that it is essentially the height of the central part of the investigated area that controls the drag, while “higher frequency” deformations, as shown for the 2nd and 3rd eigenvector in Figs. 11b and 11c, are of secondary importance.
Figure 9. (a) Gradients obtained from differentiating the 5D GPR model. (b) Eigenvalue spectrum based on the 5D GPR model gradients.

Figure 10. (a) Projection of Passat data into 1D active subspace. (b) First eigenvector.

Figure 11. Shape deformation modes corresponding to (a) first, (b) second and (c) third eigenvector. The colors indicate the magnitude of surface normal displacement.
The results of this test case of localized shape perturbations with a quasi-linear drag response of an albeit complex geometry confirm the validity of feeding the ASM with gradient data derived from a surrogate model and underpin the benefit of the ASM in terms of dimensionality reduction and physical insight. In how far these advantages carry over to a global parameterization of a realistic passenger production car will be investigated next.

### III.C. Global car shape parameterization

For the final test of this investigation, the Volkswagen aerodynamics department provided a real-life vehicle case: An 18-dimensional parameterization of the current Volkswagen Jetta (Fig. 12) along with a Latin Hypercube of 100 training and 10 test computations delivering drag, front lift and rear lift. For confidentiality reasons, we are not able to show the details of the geometry parameterization, but note that each of the 18 parameters constitutes an observable vehicle shape feature (like the length of the trunk, the inclination of the windscreen, the rear tapering angle of the vehicle etc.), and that in their entirety, the parameters give control over the whole car shape – in contrast to the localized geometric control of the previous case.

![Figure 12. Third test case: the current Volkswagen Jetta.](image)

The only 100 computations constitute a very coarse sampling in the 18-dimensional design space. As a result, when fitting a full-dimensional GPR model to the drag data (Fig. 13a), the testing and training errors reach sizeable 0.66 and 0.74%, respectively – significantly more than in the low-dimensional Passat case of the previous subsection. Based on the gradients taken from this GPR model, the eigenvalue analysis identifies a dominating one-dimensional active subspace (Fig. 13b). Projection of the drag data into this 1D subspace results, however, in a considerable spread (Fig. 14a), indicating (a) that the actual drag behavior is not one-dimensional, or (b) that the gradients from the 18D GPR model are not accurate enough to compute the first active subspace correctly. As the first eigenvector is confirmed by a linear regression of the drag data (Fig. 14b), we can rule out (b) and therefore check if the addition of further subspace dimensions increases the projection accuracy. Fig. 15 shows the evolution of the training and testing errors of GPR models built in active subspaces of increasing dimensionality. Interestingly, there is a “sweet spot” at dimensions 13 and 14, where both errors are somewhat lower not only as compared to the projection into 1D (as expected) but also in comparison to the full 18D model.

In other words, the procedure of feeding the ASM with gradients from a GPR model in the full design space allows to generate a reduced-dimension GPR model of slightly higher accuracy. When applying this procedure to the front and rear lift data of the Jetta case, we observe the same – albeit less pronounced – qualitative dependence of testing and training errors on the active subspace dimension, including – also less pronounced – “sweet spots” around dimension 8 and dimension 11, respectively (Fig. 16). A similar pattern was found by Lukaczyk\(^\text{15}\) in his application of the ASM to shape optimization of supersonic aircraft. While these consistent observations confirm the usefulness of our approach for moderate improvements of model accuracy, we also note that due to the rather flat testing error spectrum of the front and rear lift cases, these two data sets can be described adequately already by a 1D active subspace (Fig. 17), thus corroborating the potential of the ASM for dimensionality reduction of global parameterizations of real-life complex car geometries.
Figure 13. (a) 18D GPR model of the Jetta drag data. (b) Eigenvalues based on the gradients from this model.

Figure 14. (a) Projection of Jetta drag data into 1D active subspace. (b) First eigenvector.

Figure 15. Error dependence on active subspace dimension for the Jetta drag data. The dotted lines are the errors of the original 18D GPR model of Fig. 13a.
Figure 16. Error dependence on active subspace dimension for the Jetta lift data: (a) front lift, (b) rear lift. Dotted lines correspond to the errors of the full 18D GPR models for front and rear lift, respectively.

Figure 17. Projection of Jetta lift data into 1D active subspace: (a) front lift, (b) rear lift.

IV. Conclusions

Motivated on the one hand by the demand for efficient design processes in vehicle aerodynamics, and on the other by recent successes of the Active Subspace Method (ASM) in aeronautical shape optimization, we applied this method to three different parameterized bluff bodies of increasing complexity, in order to assess its benefit for dimensionality reduction in vehicle shape design. Our findings are the following:

1. Currently available adjoint implementations capable of computing sensitivities of industrially complex bluff bodies deliver gradients that are too inaccurate to justify their usage in the ASM. Instead, by using the gradients from a GPR model fitted to the available data, we devised a procedure to use the ASM when the CFD solver does not provide accurate gradients.

2. Applying this procedure to the two car test cases of this report identified dominant 1D active subspaces for drag, front and rear lift. For the Passat case, a 1D model built upon the 1D active subspace suffices to entirely describe the behavior in the overall 5D design space. For the Jetta, the 1D-active-subspace-based models capture a large fraction of the overall 18D functional dependencies on the geometric parameters. In this sense, the devised ASM-based procedure qualifies as a valid method for dimensionality reduction in vehicle design.
3. As the eigenvectors define the “shape deformation modes” of highest impact onto the function of interest, they provide physical insight into effective design changes. In contrast to adjoint-based sensitivity maps, they are valid not only for a single configuration, but represent the behavior in the entire design space.

4. With the dominant active subspace being one-dimensional in all studied cases, the results of (2) and (3) could have been obtained by simple linear regression. The added-value of the ASM in these instances is the provision of the eigenvalue spectrum, which indicates the (in)validity of the reduction to 1D. ASM obviously pays off in cases of active subspaces that are more than one-dimensional (see also 5).

5. It has been shown that the predictive accuracy of an existing GPR model can be moderately improved (at practically no additional cost) by projecting it into a suitably chosen active subspace computed from the gradients of the original GPR model.

Despite the challenges of applying the ASM in a vehicle shape design context – inaccurate adjoint-based gradients in the presence of flow separation, and the feature-based approach of vehicle parameterization – it turned out to be a useful tool: for dimension reduction, for providing physical insight and for improving the accuracy of surrogate models. The ASM will therefore be adopted as the first building block of the intended Interactive Aerodynamic Design (IAD) tool outlined above – specifically to reduce the sampling requirements for building an adequate Reduced Order Model (ROM). Future work will focus on complementing the IAD process by industrializing state-of-the-art ROM techniques.

Acknowledgments

The first author is grateful to Thomas D. Economon for numerous enlightening discussions, to Christoph Lietmeyer for providing the Jetta data set and to the members of the Aerospace Design Laboratory at Stanford University for their hospitality.

References