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Large Eddy Simulations of Flow Around a Cylinder with Uncertain Wall Heating

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Numerical simulations of the turbulent flow and heat transfer around an array of cylinder are carried out at Reynolds number $Re_D = 3,900$. The wall heat flux is assumed to be uncertain and characterized by highest variance in the stagnation point region. Simulations are carried out using a novel uncertainty propagation scheme, the hybrid stochastic projection method. This technique combines the accuracy and convergence properties of intrusive stochastic Galerkin methods with the non-intrusive nature of stochastic collocation. Reynolds averaged and Large Eddy simulations are used to estimate the variability induced by the uncertain boundary conditions; the comparison between the two approaches provides an indirect estimate of the epistemic uncertainty associated to the simplification introduced by the turbulence modeling. The sensitivity of the predictions to the assumed wall heating variability is also presented and shows that the expectation and variance of the temperature converge more slowly when LES is considered.

I. Introduction

Uncertainty quantification is a fundamental step in the process of establishing confidence in numerical predictions. Two types of uncertainties must be typically considered, the *aleatory* associated to the natural variability of the system and the *epistemic* related to lack of knowledge in the physical processes involved.

Aleatory uncertainty is naturally represented in a probabilistic framework using a set of random variables and processes with given probability distribution functions; several methods are available in the literature to *propagate* the uncertainty to the output quantity of interest. Polynomial chaos[1] (hereafter referred to as stochastic Galerkin approach) and stochastic collocation methods[2] are among the most popular; in these approaches multi-variate polynomial bases are defined on the space defined by all the input random variables. The original (deterministic) problem is therefore transformed into a *new*, high dimensional stochastic system whose solution provides the desired output of interests with the associated probability distributions.

Epistemic uncertainty, on the other hand, is not easily represented in terms of a finite set of random variables. As an example, in the analysis of complex systems it is common to use phenomenological models which are highly simplified characterization of the real physical processes; these models introduce *errors* that are difficult to identify. One approach is to build an error model based on the differences between the model predictions and actual system measurements; this *calibration* process is typically only applicable when sufficient experimental characterization of the system exists.[3] An alternative approach is to *reduce* the epistemic uncertainty by introducing more comprehensive and accurate physical models of the system, e.g. a higher fidelity representation. This approach is obviously possible if the refined models are available and feasible from a computational perspective.

The focus of the present work is to investigate the effect of uncertainties on the convective heat transfer around a bluff body. The evaluation of the heat fluxes is important in a variety of engineering applications such as the cooling passages of turbine blades; quantification of the uncertainties in the overall heating rates is crucial because modern turbine engines expose the blade components to increasingly severe thermal conditions and close to metal melting point.[4] The problem is well understood from a physics perspective and the governing equation (Navier-Stokes equations) can be solved very accurately in the laminar flow regime. As the convection velocity increases (more precisely as the Reynolds number increases) the flow becomes

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turbulent and the flow dynamics is characterized by a wide range of spatial and temporal scales, resulting in extremely high computational cost for realistic - direct - simulations. In most engineering situations only the time-averaged heat transfer is desired and, therefore, approximations can be used to simplify the problem. In particular, a phenomenological model of the turbulent fluctuations, a *turbulence model*, is introduced. In spite of several decades of efforts in developing sophisticated models based on the Reynolds-Averaged Navier-Stokes equations (RANS),[5,6] their accuracy remains largely limited. In the context of uncertainty quantification, the use of RANS models naturally leads to epistemic uncertainty, which is, as mentioned above, difficult to characterize.

The increasing computational power available using massively parallel computers provides an opportunity to capture the unsteady large scale turbulent motion and thus reducing the inaccuracy associated to RANS modeling. In the Large Eddy Simulation (LES) approach, the phenomenological model is required to to represent only the small scale fluctuations. LES has proven to provide a more realistic representation of the physical process associated to the large scale turbulent mixing (thus virtually eliminating the corresponding epistemic uncertainty). It must be stressed that the process of replacing a detailed representation of the physics (in this case the turbulence) with a model leads also to a conversion of aleatory to epistemic uncertainty; on the other hand, high fidelity simulations of turbulent flows are notoriously affected by variability in the initial and boundary conditions, thus it is expected that the predictions are sensitive to the detailed description of the aleatory uncertainty; RANS turbulence models, on the other hand, are more robust to changes in the boundary conditions.

In a previous effort we studied the impact of aleatory uncertainties associated with the wall heat flux and the inflow velocity for the flow around an array of circular cylinders using a standard RANS turbulence model. We compared various stochastic collocation methods and introduced a hybrid approach[7] which showed encouraging savings with respect to classical sampling based technique such as the Monte Carlo method. In the current work we are applying the same technique coupled to a high fidelity turbulence simulation technique, in an effort to identify the impact of epistemic uncertainty on the predictions.

In the following we first introduce the hybrid uncertainty propagation scheme and then the detailed mathematical model used in the numerical simulations.

II. Uncertainty quantification for heat transfer problems

Uncertainty quantification within a probabilistic framework starts by defining a set of random variables describing the space over which the statistics of the output of interest need to be evaluated. For the problem of interest here, we assume that the input (aleatory) uncertainty is associated with the thermal conditions on the cylinder wall and the velocity inflow conditions. From a physical point of view, we account for two missing phenomena: *i*) the heat conduction within the cylinder which might lead to an uneven temperature on its surface and *ii*) the presence of obstacles upstream of the configuration of interest that result in flow non-uniformity. We choose to represent these two uncertainty sources with two sets of random processes which can be assumed to be independent from each other because associated to completely uncorrelated physical phenomena.

The problem of interest is governed by the Navier-Stokes equations for an incompressible fluid. The evaluation of the time-averaged thermal field resulting from the cylinder wall heating (or cooling) is the main objective, and the flow velocity is required to compute the convective heat fluxes. On the other hand the fluid density is assumed to be constant, and if we ignore buoyancy, the most important effect of the temperature inhomogeneities is to change the molecular viscosity of the fluid, μ. In our previous study[7] we ignored these changes and this resulted in a *one-way* coupled system: the thermal field is a function of the velocity field but not vice versa. When the temperature variations are sufficient to affect μ the momentum and energy transport are formally coupled.

From a mathematical perspective, the problem is schematically represented by a two equation model:

$$\vec{\mathcal{L}}_u(\vec{x}, t; \vec{u}, T) = \vec{f}_u(\vec{x}, t) \tag{1}$$

$$\mathcal{L}_T(\vec{x}, t; \vec{u}, T) = f_T(\vec{x}, t) \tag{2}$$

where the notation $\vec{(\cdot)}$ represents a three-dimensional vector unless specified. The unknowns in Eq. 1-2 are the velocity field $\vec{u}$ and the temperature T although the quantity of interest is *only* the temperature distribution on specified wall surface. $\vec{\mathcal{L}}_u$ and $\mathcal{L}_T$ are the differential operators corresponding to the momentum and energy transport, respectively. Note that $\vec{\mathcal{L}}_u$ also implicitly includes the statement of mass conservation which in incompressible flows reduces to a constraint on the velocity divergence. The coupling between the equations occurs since, for example, the operator $\vec{\mathcal{L}}_u$ and forcing term $\vec{f}_u$ depend on the temperature field T.

In the present problem the velocity and the temperature field are both uncertain due to the respective boundary conditions; we consider two sets of independent random variables $\vec{Y}_u$ and $\vec{Y}_T$ with dimension d_u and d_T respectively, such that the problem becomes:

$$\vec{\mathcal{L}}_u(\vec{x}, t, \vec{Y}_u, \vec{Y}_T; \vec{u}, T) = \vec{f}_u(\vec{x}, t, \vec{Y}_u, \vec{Y}_T) \tag{3}$$

$$\mathcal{L}_T(\vec{x}, t, \vec{Y}_u, \vec{Y}_T; \vec{u}, T) = f_T(\vec{x}, t, \vec{Y}_u, \vec{Y}_T) \tag{4}$$

where the *new* random variables become additional independent variables in the problem. Stochastic Galerkin and collocation schemes attempt to solve the problem above without any additional consideration of the coupling between the velocity and thermal field. The *random* space is defined by considering the set $\vec{Y} = (\vec{Y}_u, \vec{Y}_T)$ with dimension $d = d_u + d_T$: d-variate polynomial basis are considered.

On the other hand, it is reasonable to assume that the random variables that characterize the uncertainty in one physical quantity do not affect the others except through the respective coupling terms in the governing equations; in other words the specific form of the coupling term *filters* the uncertainty. Thus, we can denote the explicit dependence in the model as:

$$\vec{\mathcal{L}}_u(\vec{x}, t, \vec{Y}_u; \vec{u}, T(\vec{Y}_T)) = \vec{f}_u(\vec{x}, t, \vec{Y}_u, T(\vec{Y}_T)) \tag{5}$$

$$\mathcal{L}_T(\vec{x}, t, \vec{Y}_T; \vec{u}(\vec{Y}_u), T) = f_T(\vec{x}, t, \vec{Y}_T) \tag{6}$$

The motivation behind this specific form of the problem is that there can be significant computational savings by taking advantage of the "dependence solely through coupling terms" to create an iterative approach to uncertainty propagation that first computes the coupling terms for one equation, then solves the equation itself.

A. One-way coupled systems

If we ignore the effect of buoyancy and the dependency of the fluid properties on temperature, the velocity field is formally decoupled from the thermal field. Uncertainties in the temperature wall conditions cannot affect the resulting flow field (they are filtered out!) while the variability in the velocity boundary conditions will change the thermal evolution of the system. This one-way coupling can be used to simplify the uncertainty propagation problem.

The mathematical problem is now:

$$\vec{\mathcal{L}}_u(\vec{x}, t, \vec{Y}_u; \vec{u}) = \vec{f}_u(\vec{x}, t, \vec{Y}_u) \tag{7}$$

$$\mathcal{L}_T(\vec{x}, t, \vec{Y}_T; \vec{u}(\vec{Y}_u), T) = f_T(\vec{x}, t, \vec{Y}_T) \tag{8}$$

and can be solved in a decoupled way, where the velocity field (and its uncertainty) is first evaluated and then the heat transfer problem under uncertain boundary conditions and velocity field is tackled. The advantage in this case is that the energy equation is linear. Note that $\vec{u} \equiv \vec{u}(\vec{x}, t, \vec{Y}_u)$ and $T \equiv T(\vec{x}, t, \vec{Y}_u, \vec{Y}_T)$. This form of the problem was considered in a previous work;[7] we used a stochastic Galerkin scheme to discretize the energy equation (in the space spanned by $\vec{Y}_T$) and stochastic collocation to characterize the uncertainty in the velocity field. In the following more details will be given.

B. Two-way coupled systems

It is possible to formulate the problem in a more general setting by retaining the coupling between the energy and the momentum equation. In this case we introduce a new set of random variables $\vec{Y}_{\hat{T}}$ which represent the uncertainty filtered by the coupling of the temperature and velocity field, with dimension $d_{\hat{T}}$. The mathematical representation of the problem is now:

$$\vec{\mathcal{L}}_u(\vec{x}, t, \vec{Y}_u; \vec{u}, T(\vec{Y}_{\hat{T}})) = \vec{f}_u(\vec{x}, t, \vec{Y}_u, T(\vec{Y}_{\hat{T}})) \tag{9}$$

$$\mathcal{L}_T(\vec{x}, t, \vec{Y}_T; \vec{u}(\vec{Y}_u), T) = f_T(\vec{x}, t, \vec{Y}_T) \tag{10}$$

where the relation between $\vec{Y}_{\hat{T}}$ and $\vec{Y}_T$ depends on the specific form of the problem. The most general case is $\vec{Y}_{\hat{T}} \equiv \vec{Y}_T$ where no filtering occurs and the full system depends on $\vec{Y} = (\vec{Y}_u, \vec{Y}_T)$; if the dimensionality of $\vec{Y}_{\hat{T}}$ is smaller than $\vec{Y}_T$ ($d_{\hat{T}} < d_T$) computational savings will result.

Several possible general choice for the parameterization of the temperature T in the momentum equation are possible:

1. Coupling based on the most likely conditions. In this case we define the expectation of the temperature as $E[T] = \int_{\vec{Y}_T} T d\rho$ and define the coupling based on $E[T]$. The system becomes:

$$\vec{\mathcal{L}}_u(\vec{x}, t, \vec{Y}_u; \vec{u}, E[T]) = \vec{f}_u(\vec{x}, t, \vec{Y}_u, E[T]) \tag{11}$$

$$\mathcal{L}_T(\vec{x}, t, \vec{Y}_T; \vec{u}(\vec{Y}_u), T) = f_T(\vec{x}, t, \vec{Y}_T) \tag{12}$$

   the momentum transfer is only coupled to the expected temperature field and therefore still formally decoupled from the uncertainty in the temperature field $\vec{Y}_T$; in other words $d_{\hat{T}} = 0$ independently of d_T.

2. Coupling based on a worst case scenario. The objective in this case is to characterize the coupling by its effect on the momentum transport; if we assume that the coupling only occurs through the change in the molecular viscosity, an increase in temperature results in a reduction of the viscosity and therefore to an increased importance of the convective fluxes over the diffusion fluxes. We can define the coupling based on $S[T] = \sup_{\vec{Y}_T} T$. The system is formally the same as in the previous case with $S[T]$ replacing the expected temperature, and the velocity field is once again formally decoupled from the set of variables $\vec{Y}_T$.

3. Coupling based on a truncated Karhunen-Loeve expansion. This is the most general case in which a new set of variables $\vec{Y}_{\hat{T}}$ is introduced to represent the variability in the temperature in the definition of the coupling term in the momentum equation.

III. Problem Description

The configuration analyzed consists of a periodic array of cylindrical pins separated by a distance $L/D = 1$ (where D is the cylinder diameter). The Reynolds number based on the incoming fluid stream (U_{ref} and D) of $Re_D = 3,900$. The problem is schematically represented in Fig. 1; we consider a periodic domain consisting on one cylinder with inflow and outflow conditions applied $5D$ upstream and $25D$ downstream of the cylinder forward stagnation point.

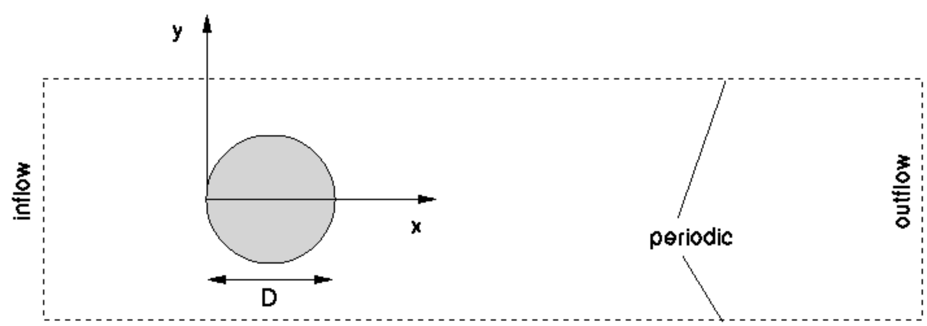

Figure 1. Sketch of the configuration used for the present numerical study.

The focus of the present investigation is to study the effect of the uncertainty on the characteristics of the heat flux on the cylinder wall and the inflow velocity. We use Reynolds Averaged Navier-Stokes modeling and Large Eddy Simulation approach to perform the numerical simulations. Given the Reynolds number considered we expect the boundary layer to be laminar as it separates from the cylinder; these conditions prove to be difficult to predict by RANS models.[8] By comparing the results obtained using both

LES and RANS we can explicitly identify the epistemic uncertainty associated to the turbulence modeling assumptions.

In the present work we used a commercial CFD code[11] to perform the RANS simulations as it was done previously;[7] we used the popular $k-\omega$.[6] The LES computations are based on an unstructured grid solver developed at Stanford.[17]

The computational grids used for the RANS and LES computations are shown in figure 2. LES requires a much finer grid with respect to the RANS simulations; in addition, the RANS computations are performed considering only a two-dimensional slice in the spanwise direction while in LES we considered a three-dimensional domain with a uniform grid in the spanwise dimension consisting on 48 cells.[8]

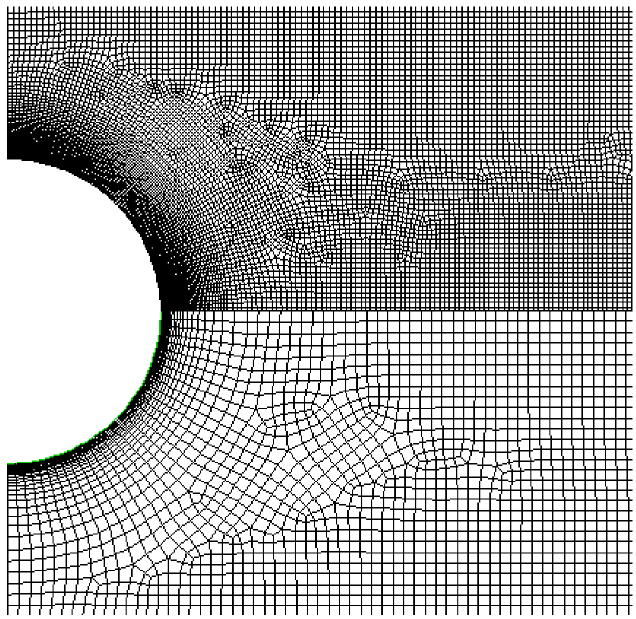

Figure 2. Unstructured grids used in the LES (top) and RANS (bottom) computations showing the strong clustering at the cylinder surface to capture the boundary layer.

We consider the inflow velocity as an *uncertain*, spatially varying function parameterized by a set of uniform random variables , $\vec{Y}_u$, with support $[-1,1]$. The inlet velocity profile is constructed as a linear combination of cosine functions of $y/D \in [-1,1]$, i.e.

$$\vec{u}_{inlet}(y, \vec{Y}_u) = \vec{u}_{ref}\left(1 + \sigma_u \sum_{k=1}^{d_u} (y_u^k \cos(k\pi y/D))\right). \tag{13}$$

where σ_u controls the amplitude of the inflow velocity fluctuations. In the present numerical experiments, we set $\sigma_u = 0.25$, which ensures that the amplitude of the random fluctuations does not cause the inlet velocity to become negative. This model allows moderate random fluctuations about a mean value, $E[\vec{u}_{inlet}] = \vec{u}_{ref}$. In our previous work[7] we considered only two terms in the sum of Eq. (13) corresponding to $k = 2$ and $k = 10$. In Fig. 3 three realization of the inflow velocity are reported.

As mentioned earlier, an additional source of uncertainty is the specification of the heat flux boundary condition on the cylinder wall. We model the heat flux boundary as an *uncertain* exponential function of $x/D \in [0,1]$ parameterized by a set of uniform random variables $\vec{Y}_T$, with support $[-1,1]$, namely

$$q_{wall}(x/D, \vec{Y}_T) = q_{ref} * exp\left[-0.1 + \sigma_T \sum_{k=1}^{d_T} y_T^k \cos(k\pi x/D))(x/D)^2\right] \tag{14}$$

where σ_T controls the influence of the uncertainty and it is set to $\sigma_T = 0.05$. In Fig. 3 we report some realizations of the heat flux and its expected value. In our previous study[7] we used a slightly different formulation and $d_T = 1$; here we consider two cases $d_T = 1$ and $d_T = 2$.

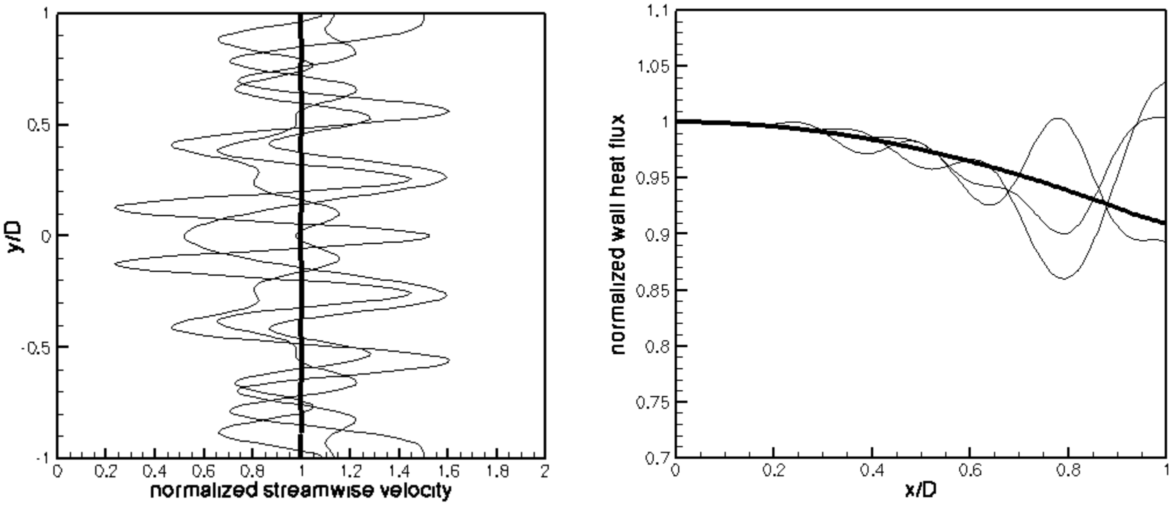

Figure 3. Inflow velocity (left) and wall heat flux (right) corresponding to three realizations (thin lines) and the expectation (thick line) from Eq. 13 and 14, respectively .

IV. Hybrid uncertainty propagation technique

In this section we briefly describe the techniques of stochastic collocation and intrusive stochastic Galerkin before introducing the present hybrid approach.[7]

A. Stochastic Collocation

Polynomial interpolation and integration of multi-variate functions are the fundamental concepts behind stochastic collocation schemes.[2] The desired statistics (integrals over the random space) of an unknown function can be approximates by evaluating it at a finite set of the parameter values (abscissa). This reduces the full stochastic problem to a set of uncoupled deterministic problems.

In the context of the systems presented in Section II, given the uncertainty in the velocity and in the thermal conditions represented by $\vec{Y}_u$ and $\vec{Y}_T$ respectively, we define a parameter space consisting of the collection of all the random variables $\vec{Y} = (\vec{Y}_u, \vec{Y}_T)$. Considering a multi-dimensional interpolation/integration formula, the solution of the Navier-Stokes equations is computed at N predefined abscissas $\vec{Y}^k$ and used to compute the statistics of the stochastic solution $(\vec{u}, T)$ with the interpolation and integration formulas. In other words, the following system:

$$\vec{\mathcal{L}}_u(\vec{x}, t, \vec{Y}^k; \vec{u}, T) = \vec{f}_u(\vec{x}, t, \vec{Y}^k) \tag{15}$$

$$\mathcal{L}_T(\vec{x}, t, \vec{Y}^k; \vec{u}, T) = f_T(\vec{x}, t, \vec{Y}^k) \tag{16}$$

is solved N times corresponding to N different abscissas $\vec{Y}^k$. This technique is called *non-intrusive*, since the computation of the statistics only involves independent deterministic solutions. Note that the integration and interpolation occur along the coordinates induced by the components of $\vec{Y}$, where the number of components d in $\vec{Y}$ gives the number of dimensions of the interpolation and integration. In the following we will compare the results obtained using the present hybrid method to a conventional stochastic collocation approach based on sparse grid construction.[13,14]

B. Stochastic Galerkin

The stochastic Galerkin technique utilizes a specific representation of the random solution - called the polynomial chaos expansion (PCE) - which corresponds to an *truncated* series of orthogonal polynomials. This representation originated with Wiener[16] and then used by Ghanem and Spanos[1] in a stochastic finite element method for structural mechanics. As in the previous approach a parameter space is defined as the sum of all the random variables $\vec{Y} = (\vec{Y}_u, \vec{Y}_T)$ and the solution is expressed as:

$$\vec{u}(\vec{x},t,\vec{Y}) = \sum_{j=0}^{\infty} \vec{u}_j(\vec{x},t)\Psi_j(\vec{Y}) \approx \sum_{j=0}^{M-1} \vec{u}_j(\vec{x},t)\Psi_j(\vec{Y}) \tag{17}$$

$$T(\vec{x},t,\vec{Y}) = \sum_{j=0}^{\infty} T_j(\vec{x},t)\Psi_j(\vec{Y}) \approx \sum_{j=0}^{M-1} T_j(\vec{x},t)\Psi_j(\vec{Y})$$

These truncated representations are inserted in the governing equations and after a Galerkin projection (in the space of the basis polynomials) a new system of equations is obtained for the coefficients $\{\vec{u}_j, T_j\}$ of the expansion:

$$\vec{\mathcal{G}}_u(\vec{x},t;\vec{u}_j,T_j) = \vec{f}_u(\vec{x},t) \tag{18}$$

$$\mathcal{G}_T(\vec{x},t;\vec{u}_j,T_j) = f_T(\vec{x},t) \tag{19}$$

where the symbol $\mathcal{G}$ is used instead of $\mathcal{L}$ to outline the fact that the Galerkin procedure changes the form of the differential operator. In particular, in the previous system the dependency on $\vec{Y}$ disappeared and the problem becomes completely deterministic.

The basis polynomials $\{\Psi_j(\vec{Y})\}$ are chosen according to the joint probability density function of $\vec{Y}$.[15] The orthogonality of $\{\Psi_j\}$ leaves a set of M coupled equations for $\{\vec{u}_j, T_j\}$. Once solved, statistics of the stochastic solution can be approximated by simple formulas of $\{\vec{u}_j, T_j\}$. While the stochastic Galerkin technique has been shown to produce highly accurate statistics, it requires that existing solvers be modified to solve for the coefficients of the expansion.

C. A hybrid propagation technique

The two methods described are combined in a hybrid technique that aims at reducing the overall computational cost of computing statistics while maintaining the accuracy. Constantine *et al.*[12] have shown the connection between stochastic collocation and stochastic Galerkin approaches in detail, here we only mention that the latter is the provably the most accurate but requires rewriting of the deterministic tools, while the former is completely non-intrusive. In the present work (as already done previously[7]) we develop a method based on the physical decoupling between the momentum equation and the energy transport; in particular, we note that the uncertainty introduced in the thermal boundary condition does not influence the velocity distribution. Since the thermal boundary conditions are a function of $\vec{Y}_T$, the velocity is therefore a function of only $\vec{Y}_u$, i.e. $\vec{u} = \vec{u}(\vec{x},t,\vec{Y}_u)$. We treat the momentum equation using a stochastic collocation scheme in the space spanned by the $\vec{Y}_u$ random variables, in other words we solve the momentum equation at N points corresponding to the quadrature abscissa $\vec{Y}_u^k$. We then define a stochastic Galerkin scheme for the temperature field where the expansion is written as:

$$T(\vec{x},t,\vec{Y}_u^k,\vec{Y}_T) \approx \sum_{j=0}^{M-1} T_j(\vec{x},t,\vec{Y}_u^k)\Psi_j(\vec{Y}_T) \tag{20}$$

The mathematical form of the problem is then:

$$\vec{\mathcal{L}}_u(\vec{x},t,,\vec{Y}_u^k;\vec{u}) = \vec{f}_u(\vec{x},t,\vec{Y}_u^k) \tag{21}$$

$$\mathcal{G}_T(\vec{x},t,\vec{Y}_u^k;\vec{u},T_j) = f_T(\vec{x},t,\vec{Y}_u^k) \tag{22}$$

where we specifically eliminated the temperature from Eq. 21 as discussed in Section II for the cases with no buoyancy and no dependency of viscosity on the temperature. dimensions.

As discussed above, the application of stochastic Galerkin technique requires the solution of a system that is different from the original deterministic one. Eq. 22 is not an exception. At each collocation point (abscissa) a set of M equations for the *modes* of the temperature expansion $T_j(\vec{x},t,\vec{Y}_u^k)$ must be solved. Given the input uncertainty the equations can be written as:

$$\frac{\partial T_j}{\partial t} + u_k\frac{\partial T_j}{\partial x_k} = \frac{\partial}{\partial x_k}\left(\kappa_{eff}\frac{\partial T_j}{\partial x_k}\right), \quad j = 0,\ldots,M-1 \tag{23}$$

where κ_{eff} is the effective thermal conductivity consisting of the molecular and the *turbulent* one.[5] The boundary conditions expresses the uncertainty and, again using Galerkin projection on Eq. (14), the flux condition for the $j-th$ mode is:

$$q^j_{wall} = \frac{E[q_{wall}\Psi_j]}{E[\Psi_j^2]} \tag{24}$$

Since the energy equation is linear and the dependence on $\vec{Y}_T$ occurs only through the boundary condition, the equations for T_j naturally decouple. Thus the problem reduces to the solution of the momentum equations and M decoupled energy-like equations at each collocation point.

To approximate the expectation and variance of the temperature and velocity fields, we compute

$$E[\vec{u}](\vec{x},t) \approx \sum_{k=0}^{N} \vec{u}(\vec{x},t,\vec{Y}_u^k)w_k \equiv \mu_u(\vec{x},t) \tag{25}$$

$$E[T](\vec{x},t) \approx \sum_{k=0}^{N} T_0(\vec{x},t,\vec{Y}_u^k)w_k \equiv \mu_T(\vec{x},t) \tag{26}$$

$$Var[\vec{u}](\vec{x},t) \approx \sum_{k=0}^{N} \vec{u}(\vec{x},t,\vec{Y}_u^k)^2 - \mu_u(\vec{x},t)^2 \tag{27}$$

$$Var[T](\vec{x},t) \approx \sum_{k=0}^{N} \left(\sum_{j=0}^{M} T_j(\vec{x},t,\vec{Y}_u^k)E[\Psi_j^2] \right) - \mu_T(\vec{x},t)^2 \tag{28}$$

V. Numerical results

The computation of the turbulent flow and hear transfer around the array of cylinder has been carried out using both a RANS model and the LES technique at a Reynolds number $Re_D = 3,900$. The results are presented in Fig. 4 and Fig. 5 for the streamwise velocity and the temperature field. Note that the RANS results represent the time-averaged solution, while in the LES computations the instantaneous (time-dependent) field are computed. The quantity of interest, namely the time-averaged wall temperature, is directly available in the RANS computations, while it needs to be explicitly evaluated from the averaging of the fluctuating fields. Fig. 5 illustrates the presence of large scale unsteadiness in the wake of the cylinder, whilst the flow is largely steady in the frontal area.

As a first step we compare the wall temperature statistics (mean and variance) obtained using different uncertainty propagation methods and RANS turbulence modeling. The boundary conditions are specified as in Eq. (13) with $d_u = 2$ and Eq. (14) with $d_T = 1$. The statistics are computed using Monte Carlo sampling (with 1,000 realizations), stochastic collocation with $d = d_u + d_T = 3$ and the present hybrid scheme with $N = 13$ (stochastic collocation in d_u) , and $M = 5$ (stochastic Galerkin expansion in d_T) for characterizing the uncertainty in the velocity and thermal fields, respectively. The excellent agreement reported in Fig. 6 is similar to what reported in our previous work[7] although in the present results the temperature variability in the stagnation point area is not very high; this is likely due to the flow remaining laminar (the Reynolds number is considerably lower). Note also that the flow separates at $x/D \approx 0.55$ and the temperature variance is substantially lower in the back portion of the cylinder.

In Fig. 7 the expected wall temperature and its variance are reported for the RANS and LES simulations performed using the hybrid uncertainty propagation scheme[a] (as before $N = 13$ and $M = 5$). The results show that the qualitative behavior of both the expected temperature and its variance is very similar; the LES and RANS predictions are very close in the vicinity of the stagnation point, but they become different approaching the separation point and then the recirculating flow region. The treatment of the turbulent motion in the LES and RANS approaches leads to different estimates of the effect of the uncertainty on the quantity of interest; this is not unexpected as mentioned earlier and it provides an *indirect* measure of the epistemic uncertainty associated to the RANS modeling - as the LES simulations have been proven to be accurate in predicting the present flow.[8] In particular from Fig. 7 we can estimate that the variance in the

[a]We have not computed the temperature statistics using LES modeling with either standard stochastic collocation or Monte Carlo because the associated computational cost is prohibitive.

wall temperature induced by the variability in the boundary conditions (aleatory uncertainty) is somewhat comparable to the variance introduced by the RANS modeling assumptions (epistemic uncertainty). A direct quantification of the epistemic uncertainty, for example using an error model[3] is outside the scope of the present paper, although it is the subject of on-going research.

The impact of the order of the Galerkin expansion on the temperature statistics has also been studied. We performed both RANS and LES simulations with a fixed stochastic collocation grid for the velocity uncertainty ($d_u = 2$ and $N = 13$) but increased the accuracy of the discretization of the thermal uncertainty, varying M from $M = 3$ to $M - 5$. The results are reported in Fig. 8 in terms of the coefficient of variation for $d_T = 1$ and $d_T = 2$[b] . The RANS results show that the statistics are not too sensitive to the expansion order and are basically already converged for $M = 3$; on the other hand, the LES computations appear to require higher order expansion to reach convergence in the case with $d_T = 1$ and are actually not converged in the case $d_T = 2$. These results illustrate that the reduction of the epistemic uncertainty obtained using the LES approach leads to an increased sensitivity to the representation of the aleatory uncertainty.

VI. Conclusions

A novel hybrid uncertainty propagation scheme has been used to study the turbulent flow and heat transfer around a bluff body subject to uncertainty in the wall heating. The method is based on a combination of stochastic collocation technique to handle the variability in the velocity field and stochastic Galerkin applied to the thermal field.

RANS and LES approaches to model turbulence have been used to identify the relative importance of epistemic and aleatory uncertainty. In the present context we assume that RANS computations correspond to a low-fidelity model. The results show that the variance associated to the epistemic uncertainty is comparable to the variance induced by the boundary conditions variability. Although the analysis was not extensive given the extreme computational effort required to quantify uncertainty in LES simulations, it is clear that approaches that enable to efficiently characterize the epistemic uncertainty are of crucial importance.

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[b]The cardinality of the multi-variate expansion used in the stochastic Galerkin method is $(M+d)!/(M!d!)$ which corresponds for $d_T = 2$ to 10, 15 and 21 terms in the expansion (20) for $M = 3$, $M = 4$ and $M = 5$ respectively.

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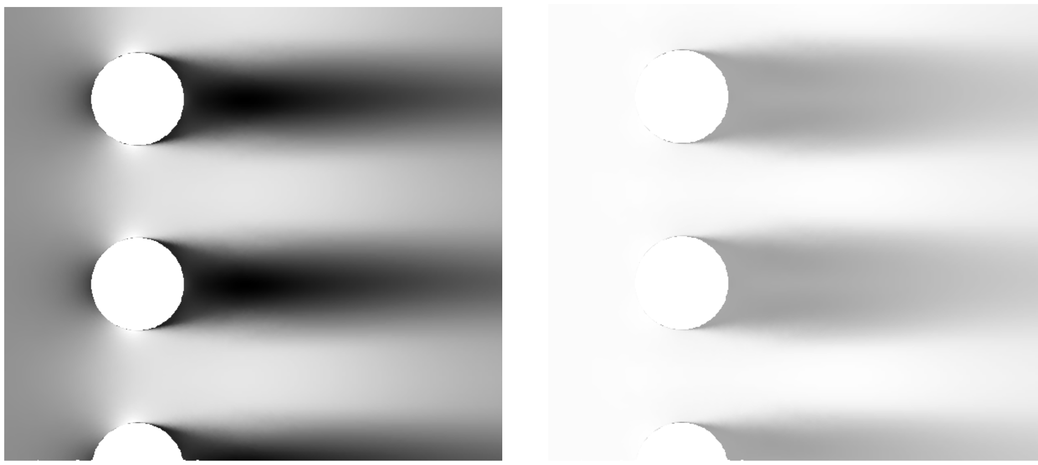

Figure 4. RANS simulations. Time-averaged velocity (left) and temperature (right) contours for the turbulent flow around an array of cylinders.

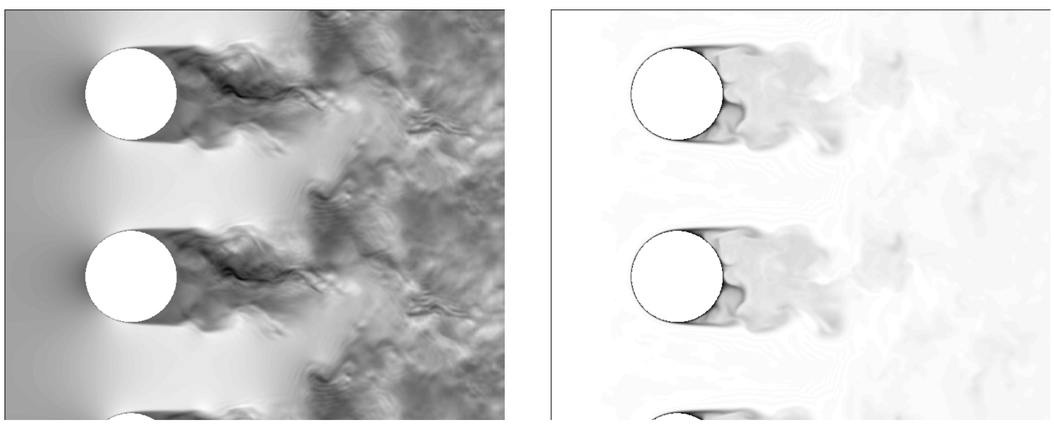

Figure 5. LES simulations. Instantaneous velocity (left) and temperature (right) contours for the turbulent flow around an array of cylinders.

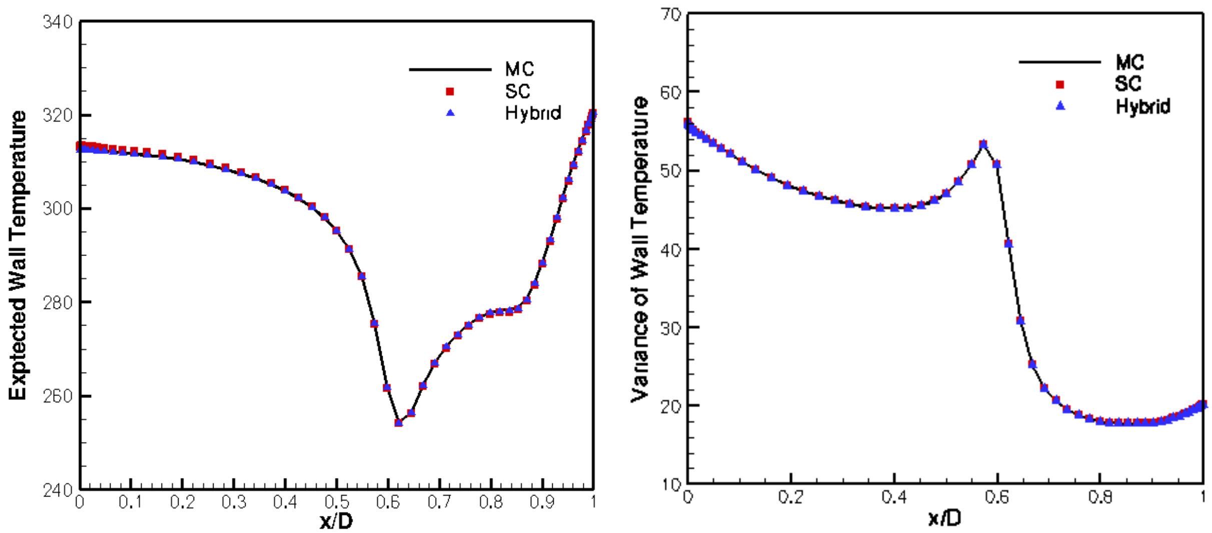

Figure 6. Temperature on the cylinder wall. Expectation (left) and variance (right) computed using RANS modeling and different uncertainty propagation methods. 1,000 samples are used in the Monte Carlo (MC) evaluation, $N = 25$ evaluations are used in the stochastic collocation (SC) method and $N = 13$ and $M = 5$ are used in the hybrid method. These calculations correspond to $d_u = 2$, $d_T = 1$.

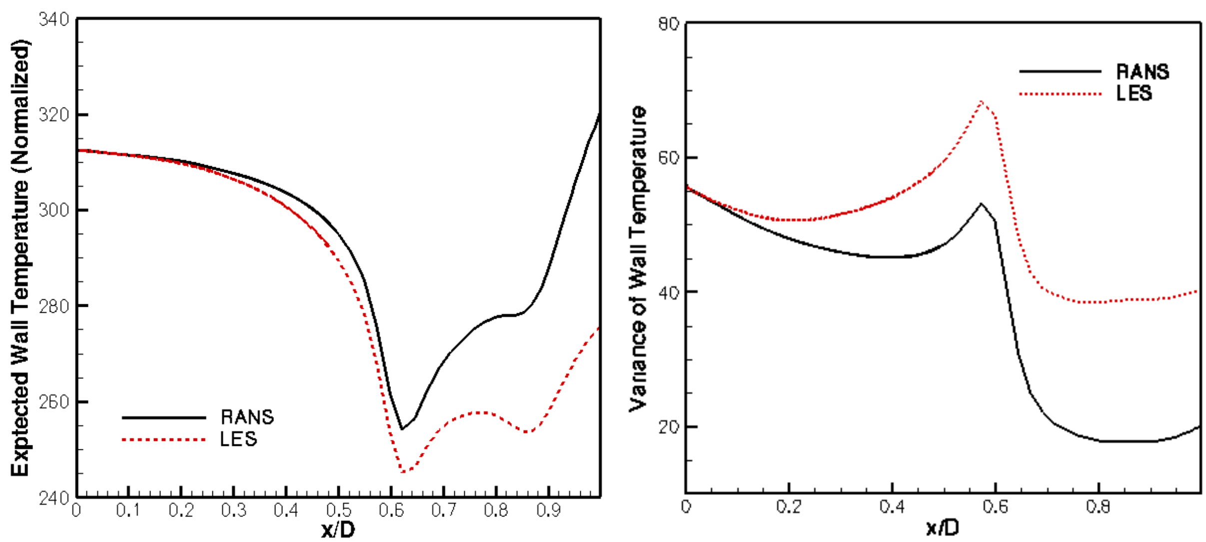

Figure 7. Temperature on the cylinder wall. Expectation (left) and variance (right) computed using RANS modeling and the LES technique. These calculations correspond to $d_u = 2$, $d_T = 1$, $N = 13$ and $M = 5$.

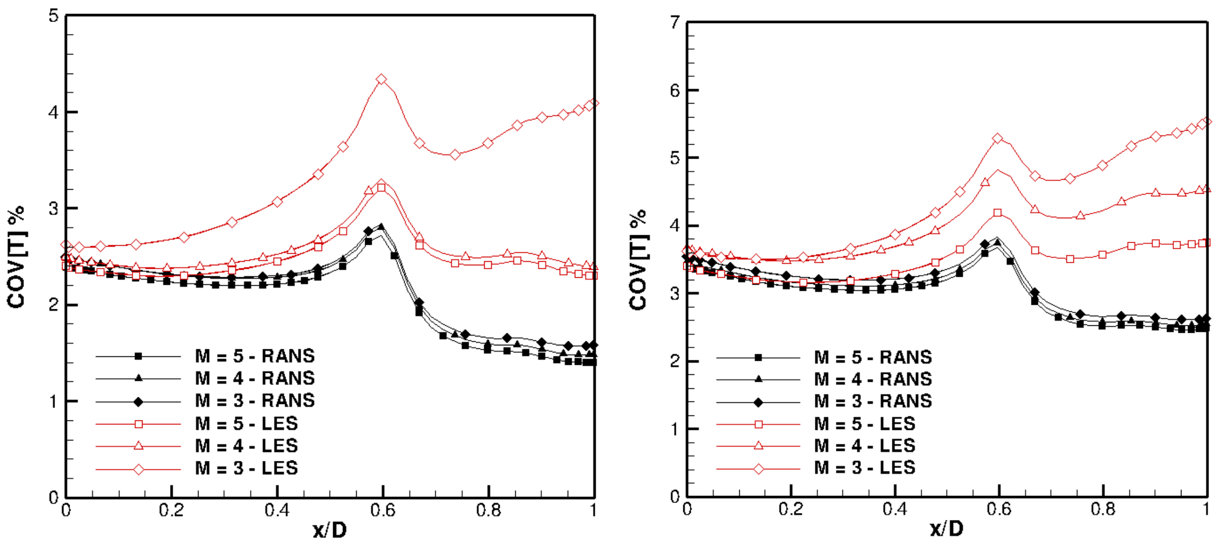

Figure 8. Coefficient of variation of the temperature on the cylinder wall. Effect of the order of the Galerkin expansion used for the energy equation for both RANS and LES modeling. The computations correspond to $d_u = 2$ and $d_T = 1$ (left) and $d_u = 2$ and $d_T = 2$ (right).