SIAM J. SCI. COMPUT.
Vol. 36, No. 6, pp. A3030–A3031

ERRATUM: ACTIVE SUBSPACE METHODS IN THEORY AND PRACTICE: APPLICATIONS TO KRIGING SURFACES*

PAUL G. CONSTANTINE†, ERIC DOW‡, AND QIQI WANG‡

Abstract. This erratum corrects the statements of Theorems 3.2, 3.3, 3.6, and 3.7 from Constantine, Dow, and Wang [*SIAM J. Sci. Comput.*, 36 (2014), pp. A1500–A1524], all of which contain a similar minor error in the application of the triangle inequality. It also corrects a missing minus sign in (5.3). These errors do not change the main conclusions of the paper.

Key words. active subspace methods, kriging, Gaussian process, uncertainty quantification, response surfaces

AMS subject classification. 65C60

DOI. 10.1137/140983598

The proofs of Theorems 3.2 and 3.3 in Constantine, Dow, and Wang [1] contain minor errors that affect the statements of the theorems. These errors propagate to the statements of Theorems 3.6 and 3.7. Here are the corrections. Equation and theorem numbers in this erratum refer to those in the original paper.

In the proof of Theorem 3.2, (3.16) should be

$$\begin{aligned}\left(\mathbb{E}\left[(f-\hat{F})^2\right]\right)^{\frac{1}{2}} &\leq \left(\mathbb{E}\left[(f-F)^2\right]\right)^{\frac{1}{2}} + \left(\mathbb{E}\left[(F-\hat{F})^2\right]\right)^{\frac{1}{2}} \\ &\leq \left(C_1\left(\lambda_{n+1}+\cdots+\lambda_m\right)\right)^{\frac{1}{2}} + \left(\frac{C_1}{N}\left(\lambda_{n+1}+\cdots+\lambda_m\right)\right)^{\frac{1}{2}} \\ &= \left(1+\frac{1}{\sqrt{N}}\right)\left(C_1\left(\lambda_{n+1}+\cdots+\lambda_m\right)\right)^{\frac{1}{2}}.\end{aligned} \tag{3.16}$$

The corrected statement of Theorem 3.2 follows.

THEOREM 3.2. *The mean-squared error of $\hat{F}$ defined in* (3.11) *satisfies*

$$\mathbb{E}\left[(f-\hat{F})^2\right] \leq C_1\left(1+\frac{1}{\sqrt{N}}\right)^2(\lambda_{n+1}+\cdots+\lambda_m), \tag{3.12}$$

where C_1 is from Theorem 3.1.

Similarly, in the proof of Theorem 3.3, (3.21) should be

$$\left(\mathbb{E}\left[(f-\tilde{F})^2\right]\right)^{\frac{1}{2}} \leq \left(\mathbb{E}\left[(f-\hat{F})^2\right]\right)^{\frac{1}{2}} + \left(\mathbb{E}\left[(\hat{F}-\tilde{F})^2\right]\right)^{\frac{1}{2}}. \tag{3.21}$$

The corrected statement of Theorem 3.3 follows.

THEOREM 3.3. *The mean-squared error in $\tilde{F}$ defined in* (3.18) *satisfies*

$$\mathbb{E}\left[(f-\tilde{F})^2\right] \leq \left(\left(C_1\left(\lambda_{n+1}+\cdots+\lambda_m\right)\right)^{\frac{1}{2}}\left(1+\frac{1}{\sqrt{N}}\right) + (C_2\,\delta)^{\frac{1}{2}}\right)^2, \tag{3.20}$$

*Submitted to the journal's Methods and Algorithms for Scientific Computing section August 25, 2014; accepted for publication September 2, 2014; published electronically December 23, 2014.
http://www.siam.org/journals/sisc/36-6/98359.html

†Department of Applied Mathematics and Statistics, Colorado School of Mines, Golden, CO 80401 (paul.constantine@mines.edu).
‡Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge, MA 02139 (ericdow@mit.edu, qiqi@mit.edu).

where C_1 is from Theorem 3.1*, N is from Theorem* 3.2*, and C_2 and δ are from Assumption* 1*.*

These errors propagate to the statements of Theorems 3.6 and 3.7, whose corrected statements follow.

THEOREM 3.6. *The mean-squared error in the Monte Carlo approximation $\hat{F}_\varepsilon$ using the perturbed eigenvectors $\tilde{\mathbf{W}}_1$ satisfies*

$$\mathbb{E}\left[(f-\hat{F}_\varepsilon)^2\right] \;\leq\; C_1\left(1+\frac{1}{\sqrt{N}}\right)^2\left(\varepsilon\,(\lambda_1+\cdots+\lambda_n)^{\frac{1}{2}} \;+\; (\lambda_{n+1}+\cdots+\lambda_m)^{\frac{1}{2}}\right)^2, \tag{3.33}$$

where C_1 and N are the quantities from Theorem 3.2*.*

THEOREM 3.7. *Under the assumptions of Theorem* 3.3*, the mean-squared error in the response surface approximation $\tilde{F}_\varepsilon$ satisfies*

$$\begin{aligned}\mathbb{E}\left[(f-\tilde{F}_\varepsilon)^2\right] \leq\; &\left(C_1^{\frac{1}{2}}\left(\varepsilon\,(\lambda_1+\cdots+\lambda_n)^{\frac{1}{2}} \;+\; (\lambda_{n+1}+\cdots+\lambda_m)^{\frac{1}{2}}\right)\left(1+\frac{1}{\sqrt{N}}\right)\right.\\ &\left.+\;(C_2\,\delta)^{\frac{1}{2}}\right)^2,\end{aligned} \tag{3.35}$$

where C_1, N, C_2, and δ are the quantities from Theorem 3.3*.*

Finally, (5.3) should have a minus sign as follows:

$$\mathcal{C}(\mathbf{s},\mathbf{t}) \;=\; \exp\left(-\beta^{-1}\,\|\mathbf{s}-\mathbf{t}\|_1\right). \tag{5.3}$$

REFERENCE

[1] P.G. Constantine, E. Dow, and Q. Wang, *Active subspaces in theory and practice: Applications to kriging surfaces*, SIAM J. Sci. Comput., 36 (2014), pp. A1500–A1524.