ERRATUM: ACTIVE SUBSPACE METHODS IN THEORY AND PRACTICE: APPLICATIONS TO KRIGING SURFACES*

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Abstract. This erratum corrects the statements of Theorems 3.2, 3.3, 3.6, and 3.7 from Constantine, Dow, and Wang [SIAM J. Sci. Comput., 36 (2014), pp. A1500–A1524], all of which contain a similar minor error in the application of the triangle inequality. It also corrects a missing minus sign in (5.3). These errors do not change the main conclusions of the paper.

Key words. active subspace methods, kriging, Gaussian process, uncertainty quantification, response surfaces

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The proofs of Theorems 3.2 and 3.3 in Constantine, Dow, and Wang [1] contain minor errors that affect the statements of the theorems. These errors propagate to the mean-squared error of Theorem 3.2. Similarly, in the proof of Theorem 3.3, (3.21) should be

\[ \left( \mathbb{E} \left[ (f - \hat{F})^2 \right] \right)^{\frac{1}{2}} \leq \left( \mathbb{E} \left[ (f - F)^2 \right] \right)^{\frac{1}{2}} + \left( \mathbb{E} \left[ (F - \hat{F})^2 \right] \right)^{\frac{1}{2}} \]

The corrected statement of Theorem 3.2 follows.

THEOREM 3.2. The mean-squared error of \( \hat{F} \) defined in (3.11) satisfies

\[ \mathbb{E} \left[ (f - \hat{F})^2 \right] \leq C_1 \left( 1 + \frac{1}{\sqrt{N}} \right)^2 (\lambda_{n+1} + \cdots + \lambda_m), \]

where \( C_1 \) is from Theorem 3.1.

Similarly, in the proof of Theorem 3.3, (3.21) should be

\[ \left( \mathbb{E} \left[ (f - \hat{F})^2 \right] \right)^{\frac{1}{2}} \leq \left( \mathbb{E} \left[ (f - F)^2 \right] \right)^{\frac{1}{2}} + \left( \mathbb{E} \left[ (F - \hat{F})^2 \right] \right)^{\frac{1}{2}}. \]

The corrected statement of Theorem 3.3 follows.

THEOREM 3.3. The mean-squared error of \( \hat{F} \) defined in (3.18) satisfies

\[ \mathbb{E} \left[ (f - \hat{F})^2 \right] \leq \left( C_1 (\lambda_{n+1} + \cdots + \lambda_m) \right)^{\frac{1}{2}} \left( 1 + \frac{1}{\sqrt{N}} \right) + (C_2 \delta)^{\frac{1}{2}}, \]

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where $C_1$ is from Theorem 3.1, $N$ is from Theorem 3.2, and $C_2$ and $\delta$ are from Assumption 1.

These errors propagate to the statements of Theorems 3.6 and 3.7, whose corrected statements follow.

**Theorem 3.6.** The mean-squared error in the Monte Carlo approximation $\hat{F}_\varepsilon$ using the perturbed eigenvectors $\tilde{W}_1$ satisfies

$$
\mathbb{E} \left[ (f - \hat{F}_\varepsilon)^2 \right] \leq C_1 \left( 1 + \frac{1}{\sqrt{N}} \right)^2 \left( \varepsilon (\lambda_1 + \cdots + \lambda_n)^{\frac{1}{2}} + (\lambda_{n+1} + \cdots + \lambda_m)^{\frac{1}{2}} \right)^2,
$$

where $C_1$ and $N$ are the quantities from Theorem 3.2.

**Theorem 3.7.** Under the assumptions of Theorem 3.3, the mean-squared error in the response surface approximation $\tilde{F}_\varepsilon$ satisfies

$$
\mathbb{E} \left[ (f - \tilde{F}_\varepsilon)^2 \right] \leq \left( C_1^2 \left( \varepsilon (\lambda_1 + \cdots + \lambda_n)^{\frac{1}{2}} + (\lambda_{n+1} + \cdots + \lambda_m)^{\frac{1}{2}} \right) \left( 1 + \frac{1}{\sqrt{N}} \right) + (C_2 \delta^{\frac{1}{2}})^2 \right)^2,
$$

where $C_1$, $N$, $C_2$, and $\delta$ are the quantities from Theorem 3.3.

Finally, (5.3) should have a minus sign as follows:

$$
(5.3) \quad C(s, t) = \exp \left( -\beta^{-1} ||s - t||_1 \right).
$$

REFERENCE