

# CSCI 5582

## Artificial Intelligence

Lecture 12  
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## Today 10/12

- Review
- Basic probability
- Break
- Belief Networks

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## Review

- Where we are...
  - Agents can use search to find useful actions based on looking into the future
  - Agents can use logic to complement search to represent and reason about
    - Unseen parts of the current environment
    - Past environments
    - Future environments
  - And they can play a mean game of chess

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## Where we aren't

- Agents can't
  - Deal well with uncertain situations (not clear people are all that great at this)
  - Learn
  - See, speak, hear, move, or feel

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## Exercise

- You go to the doctor and for insurance reasons they perform a test for a horrible disease
- You test positive
- The doctor says the test is 99% accurate
- Do you worry?

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## An Exercise

- It depends; let's say...
  - The disease occurs 1 in 10000 folks
  - And that the 99% means that 99 times out a 100 when you give the test to someone without the disease it will return negative
  - And that when you have the disease it always says you are positive
  - Do you worry?

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## An Exercise

- The test's **false positive rate** is 1/100
- Only 1/10000 people have the disease
- If you gave the test to 10000 random people you would have
  - 100 false positives
  - 1 true positive
- Do you worry?

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## An Exercise

- Do you worry?
  - Yes, I always worry
  - Yes, my chances of having the disease are 100x they were before I went to the doctor
    - Went from 1/10000 to 1/100 (approx)
  - No, I live with a lot of other 1/100 bad things without worrying

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## Another Example

- You hear on the news...
  - People who attend grad school to get a masters degree have a 10x increased chance of contracting schistosomiasis
- Do you worry?
  - Depends on where you go to grad school

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## Back to Basics

- Prior (or unconditional) probability
  - Written as  $P(A)$
  - For now think of  $A$  as a proposition that can turn out to be True or False
  - $P(A)$  is your belief that  $A$  is true given that you know nothing else relevant to  $A$

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## Also

- Just as with logic we can create complex sentences with a partially compositional semantics (sort of)...

$$P(A \wedge B), P(A \vee B), P(\neg A \vee B)...$$

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## Basics

- Conditional (or posterior) probabilities
- Written as  $P(A|B)$
- Pronounced as the probability of  $A$  given  $B$
- Think of it as your belief in  $A$  given that you know absolutely that  $B$  is true.

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## And

- $P(A|B)$ ... your belief in  $A$  given that you know  $B$  is true
- **AND**  $B$  is **all** you know that is relevant to  $A$

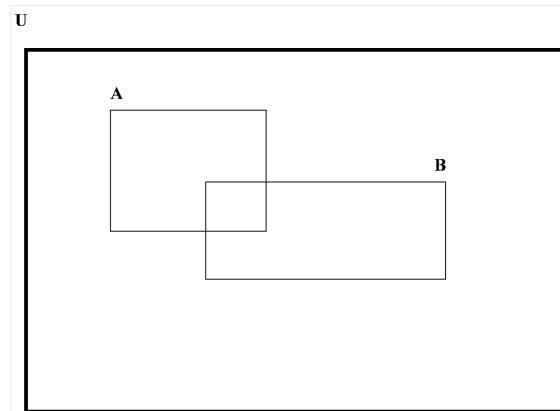
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## Conditionals Defined

- Conditionals 
$$P(A|B) = \frac{P(A \wedge B)}{P(B)}$$
- Rearranging 
$$P(A \wedge B) = P(A|B)P(B)$$
- And also 
$$P(A \wedge B) = P(B|A)P(A)$$

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## Conditionals Defined



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## Inference

- Inference means updating your beliefs as evidence comes in
  - $P(A)$ ... belief in A given that you know nothing else of relevance
  - $P(A|B)$ ... belief in A once you know B and nothing else relevant
  - $P(A|B \wedge C)$  belief in A once you know B and C and nothing else relevant

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## Also

- What you'd expect... we can have  $P(A|B \wedge C)$  or  $P(A \wedge D|E)$  or  $P(A \wedge B|C \wedge D)$  etc...

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## Joint Semantics

- Joint probability distribution... the equivalent of truth tables in logic
- Given a complete truth table you can answer any question you want
- Given the joint probability distribution over  $N$  variables you can answer any question you might want to that involve those variables

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## Joint Semantics

- With logic you don't always need the whole truth table; you can use inference methods and compositional semantics
  - I.e if I know the truth values for  $A$  and  $B$ , I can retrieve the value of  $A \wedge B$
- With probability, you need the joint to do inference unless you're willing to make some assumptions

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## Joint

	Toothache=True	Toothache=False
Cavity = True	0.04	0.06
Cavity = False	0.01	0.89

- What's the probability of having a cavity and a toothache?
- What's the probability of having a toothache?
- What's the probability of not having a cavity?
- What's the probability of having a toothache or a cavity?

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## Note

- Adding up across a row is really a form of reasoning by cases...
- Consider calculating  $P(\text{Cavity})$ ...
  - We know that in this world you either have a toothache or you don't. I.e. toothaches **partition** the world.
  - So...

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## Partitioning

$$P(\text{Cavity}) = P(\text{Cavity} \wedge \text{Toothache}) \\ + P(\text{Cavity} \wedge \neg \text{Toothache})$$

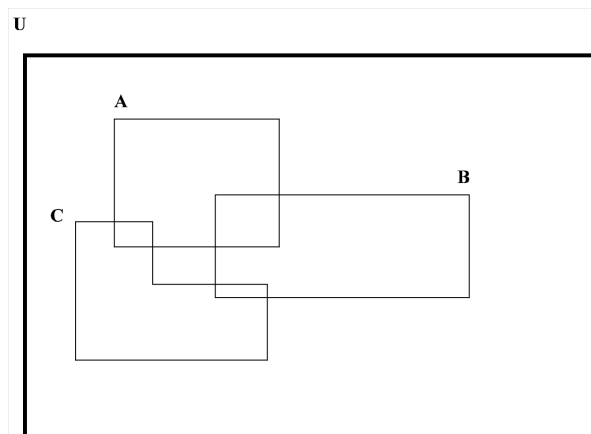
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## Combining Evidence

- Suppose you know the values for
  - $P(A|B)=0.2$
  - $P(A|C)=0.05$
  - Then you learn B is true
    - What's your belief in A?
  - Then you learn C is true
    - What's your belief in A?

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## Combining Evidence



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## Details...

- Where do all the numbers come from?
  - Mostly counting
  - Sometimes theory
  - Sometimes guessing
  - Sometimes all of the above

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## Numbers

- $P(A)$  
$$\frac{\text{Count}(\text{All } A\text{s})}{\text{Count}(\text{All Events})}$$
- $P(A \wedge B)$  
$$\frac{\text{Count}(\text{All } A \text{ and } B \text{ together})}{\text{Count}(\text{All Events})}$$
- $P(A|B)$  
$$\frac{\text{Count}(\text{All } A \text{ and } B \text{ Together})}{\text{Count}(\text{All } B\text{s})}$$

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## Break

- HW Questions?

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## Bayes

- We know...

$$P(A \wedge B) = P(A | B)P(B)$$

*and*

$$P(A \wedge B) = P(B | A)P(A)$$

- So rearranging things

$$P(A | B)P(B) = P(B | A)P(A)$$

$$P(A | B) = \frac{P(B | A)P(A)}{P(B)}$$

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# Bayes

- Memorize this

$$P(A | B) = \frac{P(B | A)P(A)}{P(B)}$$

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# Bayesian Diagnosis

- Given a set of symptoms choose the **best** disease (the disease most likely to give rise to those symptoms)
  - I.e. Choose the disease the gives the highest  $P(\text{Disease} | \text{Symptoms})$  for all possible diseases
- But you probably can't assess that...
- So maximize this...

$$P(\text{Disease} | \text{Symptoms}) = \frac{P(\text{Symptoms} | \text{Disease})P(\text{Disease})}{P(\text{Symptoms})}$$

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## Meningitis

$$P(S | M) = 0.5$$

$$P(M) = 0.00002$$

$$P(S) = 0.05$$

so....

$$P(M | S) = \frac{P(S | M)P(M)}{P(S)}$$

$$= \frac{0.5 * 0.00002}{0.05}$$

$$= 0.0002$$

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## Differential Diagnosis

- Why on earth would anyone know  $P(S)$ ?
- And do you need to know it?
  - Asking for the most probable disease given some symptoms doesn't entail knowing the probability of the diseases.

$\text{Argmax}_D P(D|S) = P(S|D) P(D)/P(S)$  is the same as

$$\text{Argmax}_D P(D|S) = P(S|D) P(D)$$

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## Well

- What if you needed the exact probability

$$\begin{aligned}P(S) &= P(S \wedge M) + P(S \wedge \neg M) \\ &= P(S | M)P(M) + P(S | \neg M)P(\neg M)\end{aligned}$$

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## Next Time

- Graphical models or Belief Nets
  - Chapter 14
- Quiz is postponed 1 Week
  - Now on 10/26
  - Covers 7, 8, 9, 13 and 14

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