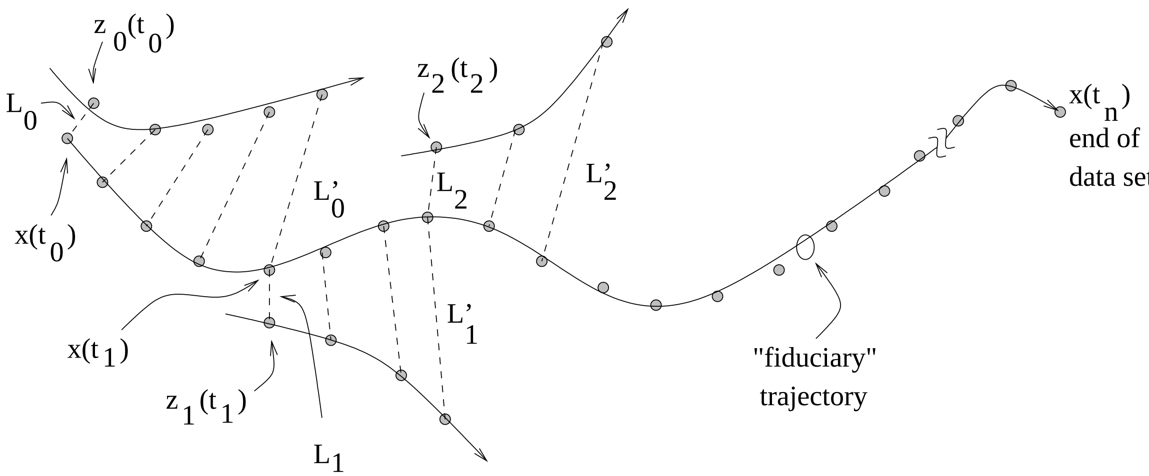
University of Colorado
Department of Computer Science

Chaotic Dynamics – CSCI 4446/5446

Wolf's algorithm for computing Lyapunov exponents from data:



Algorithm:

1. Embed the data set.

2. Pick a point $x(t_0)$ somewhere in the middle of the trajectory (but closer to the beginning is good).

3. Find that point's nearest neighbor. Call that point $z_0(t_0)$.

4. Compute $||z_0(t_0) - x(t_0)|| = L_0$.

5. Follow the "difference trajectory" — the dashed line — forwards in time, computing $||z_0(t_i) - x(t_i)|| = L_0(i)$ and incrementing i, until $L_0(i) > \epsilon$. Call that value L'_0 and that time t_1.

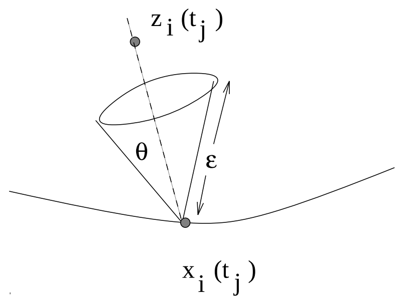
6. Find $z_1(t_1)$, the "nearest neighbor" of $x(t_1)$, and loop to step 4. Repeat the procedure to the end of the fiduciary trajectory ($t = t_n$), keeping track of the L_i and L'_i.

Use this formula to compute λ_1, the biggest (positive) Lyapunov exponent:

$$\lambda_1 \approx \frac{1}{N\Delta t} \sum_{1}^{M-1} log_2 \frac{L'_i}{L_i}$$

...where M is the number of times you went through the loop above, and N is the number of timesteps in the fiduciary trajectory. $N\Delta t = t_n - t_0$.

Schematic of the directional "nearest neighbor" computation:



Some hints:

- You'll need to come up with some sort of metric to balance nearness (how close the point is to $x_i(t_j)$) and directionality (how close the point is to the vector from $x_i(t_j)$ to $z_i(t_j)$...the dashed line in the schematic above). That choice is up to you.
- Start with $\theta = \pi/9$ and increase if necessary.
- If your data set contains transients, that can mess up this algorithm
- Short fiduciary trajectories can also mess things up
- If your program can't find a neighbor within ϵ, it should stop and report that fact to the user (who should use that information to adjust ϵ for the next run).

References:

- P. Bryant *et al.*, "Lyapunov exponents from observed time series," *Physical Review Letters*, **65**:1523-6 (1990).
- J.-P. Eckmann *et al.*, "Liapunov exponents from time series," *Phys. Rev. A* **34**:4971 (1986).
- M. Sano, "Measurement of the Lyapunov Exponents from a Chaotic Series," *Physical Review Letters* **55**:1082-1085 (1983).
- A. Wolf, "Quantifying chaos with Lyapunov exponents," in *Chaos*, Princeton University Press, 1986.

Please see the "Lyapunov exponents" section of the "Reading assignments for PS8-10 (Nonlinear Time Series Analysis)" handout.