University of Colorado
Department of Computer Science

Chaotic Dynamics – CSCI 4446/5446
Spring 2019

Problem Set 2

Issued:	22 January 2019
Due:	29 January 2019

Reading: *Strogatz, chapter 10 (except section 10.7).* Chapter 8 in Parker&Chua has some helpful pieces, but parts of it are quite complex. See also chapter 4 of Baker&Gollub. Both books are on reserve in the Engineering Library.

Online assignment: Tuesday: unit 2.1 and 2.2 videos. Thursday: unit 2.3, 2.4, and 4.2 videos. Friday: quizzes 2.1, 2.3, and 4.2

Bibliography:

- R. Abraham and C. Shaw, *Dynamics: The Geometry of Behavior*, 2nd edition (this is important!), Addison Wesley, 1992. This is out of print, but is on library reserve for this course.
- R. L. Devaney and L. Keen, eds., *Chaos and Fractals: The Mathematics Behind the Computer Graphics*, American Mathematical Society, 1988 (Proceedings of Symposia in Applied Mathematics, volume 39).
- M. J. Feigenbaum, "Universal Behavior in Nonlinear Systems," *Los Alamos Science* **1**:4-27, 1980. In the Cvitanovic reprint collection, which is on library reserve for this course.
- B.-L. Hao, "Symbolic Dynamics and Characterization of Complexity," *Physica D* **51**:161-176 (1991). This article is in the Campbell reprint collection, which is on library reserve for this course.
- M. Hénon, "A Two-Dimensional Mapping with a Strange Attractor," *Comm. Math. Physics* **50**:69-77 (1976). Reprinted in the Cvitanovic collection, which is also on library reserve for this course.
- T-Y. Li and J. A. Yorke, "Period Three Implies Chaos," *American Mathematical Monthly*, **82**:985-992, 1975. This is the paper that gave the field its name.
- N. Metropolis, M. L. Stein, and P. R. Stein, "On Finite Limit Sets for Transformations of the Unit Interval," *J. Combinational Theory* **15**:25-44 (1973). Also in Cvitanovic's collection (see above).

- S. Smale, "Differentiable Dynamic Systems," *Bull. AMS* **74**:747 (1964).
- J. M. T. Thompson and H. B. Stewart, *Nonlinear Dynamics and Chaos*, Wiley, 1986.

Problems:

1. Write a program that displays m iterates of the logistic equation on the axes x_n versus R (a "bifurcation plot"), but add an argument l that allows the user to suppress the plotting of the first l points (the transient). Turn in a bifurcation plot for the range $2.8 < R < 4$. Pick l, m, and the interval between R values such that the details of the behavior are visible, but not such that the plot requires exorbitant amounts of CPU time. You'll need to use small point icons to get this to look reasonable—no diamonds ($\diamondsuit$) or other symbols, just dots ($\cdot$)!

As always in this course, the plots you turn in should be what *your* code produces, not screenshots of some app from the web.

2. Expand the first period-doubling cascade on your plot from problem 1, zeroing in on the bifurcation points, and estimate the Feigenbaum number experimentally. (You can get much better resolution on this by looking at the numbers in the iteration sequence than by looking at the graph, by the way, and you'll need that resolution when you get into the smaller-width bifurcations.) Use at least the first four bifurcations to compute your estimates and remember that these estimates will get closer to the true 4.66... value as you get deeper into the bifurcation tree.

3. Repeat problem 2 with the Hénon map:

$$\begin{aligned} x_{k+1} &= y_k + 1 - a x_k^2 \\ y_{k+1} &= b x_k \end{aligned}$$

Use $b = 0.3$ and $0 < a < 1.4$. See page 208 of Parker and Chua if you're interested in learning more about this map. Please also turn in a plot of the bifurcation diagram of this map.

4. Are your answers to problems 2 and 3 the same (or close)? *Should* they be the same?