University of Colorado
Department of Computer Science

Chaotic Dynamics – CSCI 4446/5446
Spring 2018

Problem Set 13

Issued:	16 April 2018
Due:	30 April 2018

Note: only undergraduates are required to turn in this problem set, but *all* students should do the reading listed below.

Graduate Students: final papers are due **in pdf format** by 5pm on 3 May 2018 to `lizb@colorado.edu`.

Reading: finish the *Classical Mechanics notes;* skim the Hut & Bahcall paper listed below if you're interested in the binary star interaction.

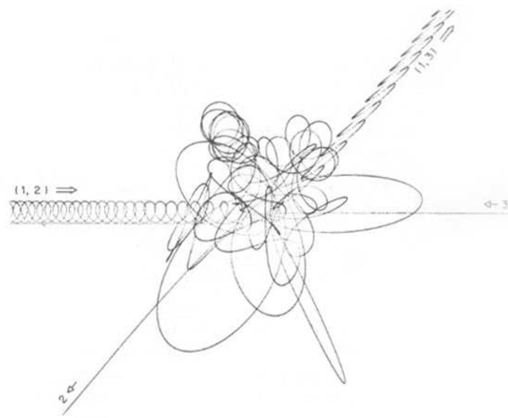
Online assignment: Tuesday 4/23 and Thursday 4/25: no videos. Tuesday 4/30: unit 9.3 video. Thursday 5.1: unit 4.5 and 10.2 videos. Friday 5/2: quizzes 9.3 and 10.2.

Bibliography:

- T. Bass, *The Eudaemonic Pie*, Penguin, 1985. On library reserve. Also published as *The Newtonian Casino.*
- D. Berreby, "Chaos hits Wall Street," *Discover*, March 1993.
- M. Casdagli, "Nonlinear prediction of chaotic time series," *Physica D* **35:**335-356 (1989).
- M. Casdagli and S. Eubank, eds., *Nonlinear Modeling and Forecasting*, Addison Wesley, 1992. On library reserve.
- J. Farmer and J. Sidorowich, "Predicting chaotic time series," *Physical Review Letters* **59**:845-848 (1987).
- J. Guckenheimer, "Noise in Chaotic Systems," *Nature* **298**:358-361 (1982).
- P. Holmes, "Nonlinear oscillations and the Smale horseshoe map," in *Chaos and Fractals: The Mathematics Behind the Computer Graphics*, American Mathematical Society, 1988 (Proceedings of Symposia in Applied Mathematics, volume 39).
- P. Hut and J. Bahcall, "Binary-single star scattering. I: Numerical experiments for equal masses," *Astrophysical Journal* **268**:319-341 (1983).
- E. Ott *et al*, "Controlling chaos," *Physical Review Letters* **64**:1196-1199 (1990).
- T. Shinbrot, "Progress in the Control of Chaos," *Advances in Physics* **44**:73-111 (1995).

Problems:

At the top of the following page is a picture of an interesting interaction between three stars.



This image originally came out in the Hut & Bahcall paper listed above, and was reprinted in the Binney & Tremaine book. Your task in this problem set is to duplicate this picture. You derived initial conditions for the binary — stars 1 and 2 in the picture — in PS11 and integrated its equations in PS12. For this set, you just have to add the third star (labeled "3" in the picture).

1. First, build a Runge-Kutta implementation of the three-body equations (the system (6) in the *Classical Mechanics Notes*), using the same type of decomposition/rewriting/reduction process that you used to get the two-body equations in PS12. For this problem set, an adaptive integrator really would be much better, as the timescales will change rapidly, but a fixed-timestep version will work too.

Test your program using the initial conditions that you derived for the PS11/12 binary for two of the stars and starting the third star, of equal mass and at rest, several thousand units away along the y axis[1]. The binary should orbit as it did on PS12 if the third star is far enough away, right? (Hint: look carefully at the picture above.)

2. In the Hut & Bahcall picture, the "field star" — whose mass is equal to that of each of the other two stars — sails into the binary from the right, along its centerline.

Start the binary at the initial conditions you derived in PS11/12 and the field star 20 units away on the positive y axis, moving directly ($\dot{x} = \dot{z} = 0$) towards the binary at 0.15 units per second.

Run an integration of the three-body equations with these initial conditions. Plot the results in *physical* space, as in PS12, looking down from above on the plane of the orbit. Does your picture look like the Hut&Bahcall picture? Any ideas on why, if not? (Hint: think frames of reference...)

3. Play with the true anomaly — the orientation of the binary when the field star "hits" it — and describe the interactions you see. (Hint: rather than computing new initial conditions for each new orientation of the binary, just start the field star further away. This isn't quite what Hut and Bahcall did, but it has almost the same effects.)

Ideally, you'll duplicate the exchange seen in the picture, since I copied all of the numbers from the original paper, but there are lots of other interesting things that can happen.

[1] Recall that the semimajor axis of the binary's orbit was one unit.