

$$T(n) = \begin{cases} 1 & n = 1 \\ aT(n/b) + D(n) & n > 1, n \text{ a power of } b \end{cases}$$

**F Master Theorem.** For any nondecreasing function  $f(n)$  and any  $d \geq 0$ ,

$$T(n) = \begin{cases} \Theta(D(n)) & D(n) = \Theta(n^d f(n)) & h < d \\ O(D(n) \log n) & D(n) = \Theta(n^h f(n)) \\ \Theta(n^h) & D(n) = O(n^d) & h > d \end{cases}$$

3. the middle case is tight, i.e.,  $T(n) = \Theta(D(n) \log n)$  for  $D(n) = \Theta(n^h f(n))$ , if  $f(n)$  satisfies this “flatness condition”:

$$(F) \quad f(\sqrt{n}) = \Omega(f(n))$$

e.g.,  $f(n) = \log n$  satisfies (F),  $f(n) = n$  doesn't

the set of  $f$ 's satisfying (F) is closed under product, powers, logs

e.g.,  $\log^2 n$ ,  $\sqrt{\log n}$ ,  $\log \log n$  satisfy (F)

we can also relax (F), requiring it only for sufficiently large  $n$

**F Master Theorem for Unequal Subproblems.**

Consider any recurrence

$$T(n) = \sum_{i=1}^k T(a_i n) + D(n)$$

where  $0 < a_i < 1$ ,  $i = 1, \dots, k$  and  $D(n) = n^d f(n)$  for a nondecreasing function  $f(n)$ .  
(change the arguments  $a_i n$  to  $a_i n + A_i$  if you wish).

Set  $s = \sum_{i=1}^k a_i^d$ .

$$T(n) = \begin{cases} \Theta(D(n)) & s < 1 \\ O(D(n) \log n) & s = 1 \\ \Theta(n^h) & s > 1 \end{cases}$$

where  $h$  satisfies  $\sum_{i=1}^k a_i^h = 1$ .

The middle case is tight,  $T(n) = \Theta(D(n) \log n)$ , for  $s = 1$  and  $f$  satisfying (F).