5. Assessment of Children's Spatial Learning

5.1 Overview

Students' spatial learning was studied for four years across different age groups and modes of representation. This chapter offers a summary and analysis of students' performance on (1) tasks of matching folding nets to developed solids; (2) the types of terms children use to describe three-dimensional shapes; and (3) students' drawings of three-dimensional objects and two-dimensional folding nets. Results from two standardized tests of spatial thinking are also discussed. The first standardized test is a near-transfer test of surface development and the second is a far-transfer test of mental rotation.

5.1.1 Assessment goal

The purpose of this series of assessment activities was to determine whether work with HyperGami and JavaGami helps students to develop more sophisticated ways of thinking about, parsing, describing, and deconstructing three-dimensional shapes and their corresponding folding nets.

5.1.2 Phases

Preliminary field work took place with the students using HyperGami in Fall 1994 and Spring 1995. The goals of the early field work phase were: (1) to gain a rough sense of whether there was any spatial learning taking place as a result of using the software; (2) to gain a better understanding of appropriate ways of assessing different types of spatial reasoning skills that might be affected by the software; (3) to determine the appropriate level of difficulty for assessment tasks, and (4) to gain general practice in working with children on the software and to come to an understanding of the types of activities that were both suitable and engaging for different age ranges.

General pilot studies and spatial assessment in HyperGami took place with elementary- and middle-school students from Spring 1995 to the end of Spring 1997. Work with high school students in a classroom setting took place from Spring 1997 through the middle of Fall 1998. A small group of students worked only with paper shapes -- that is, they built polyhedra from folding nets generated by software but did not use the software itself -- in Fall 1997. Finally, a small pilot study took place with elementary- and middle-school children using JavaGami software in Fall 1998.

Tables 5-1 and 5-2 below show the distribution of students taking part in different phases of assessment. A total of 78 elementary-, middle-, and high-school students worked with the systems from Fall 1994 to Fall 1998. The students were instructed and tested by the author or by the author in collaboration with M. Eisenberg. Numbers in parentheses indicate students who continued work with the system from a previous time period.

Table 5-1. Elementary and middle-school students working with HyperGami and JavaGami, 1994-1998.

group	time period	total	m	f	ages	system used
HG1	Fall 1994	6	2	4	grades 3-8	Early HyperGami
HG2	Spring 1995	4 (+2)	2 (+2)	2	grades 5 & 7	HyperGami
HG3	Fall 1995 to Spring 1996	7	2	5	grades 3 & 5	HyperGami
HG4	Fall 1996 to Spring 1997	4	2	2	grades 4, 6, 7	HyperGami
Shapes	Fall 1997	6	2	4	grades 2, 5, 6	Paper Shapes Only
JavaGami	Fall 1998	5 (+2)	4	1 (+2)	grades 5-8	JavaGami
	Total elem. & middle-school	32	14	18		

Table 5-2. High school students working with HyperGami, 1997-1998.

group	time period	total	m	f
HS1	Feb.- March 1997	3	3	0
HS2	April - May 1997	5	5	0
HS3	October 1997	6	0	6
HS4	Nov. - Dec. 1997	8	4	4
HS5	Feb.-March 1998	8	4	4
HS6	April-May 1998	3	1	2
HS7	Sept.-Oct. 1998	13 (+1)	8 (+1)	5
	Total hs students	46	25	21

5.1.3 Assessment chart

Table 5-3 below is a summary of the types of assessment used. Appendix E contains sample protocols and questions for the shape interview, solid drawing, and net drawing tasks.

Table 5-3. Assessment summary descriptions.

type of assessment	description	when used
net and solid card matching	students are shown folding nets and are asked to select the corresponding polyhedra from a set of paper shapes	• HG3 (F95 and S96)
shape interview	students verbally describe polyhedra	• HG2-HG4 (S95-S97) • Shapes (F97) • JavaGami (F98)
solid drawing	students draw pictures of paper polyhedra	• HG4 (F96-S97) • HS1 -HS7 (S97-F98) • Shapes (F97) • JavaGami (F98)
net drawing	students are given paper polyhedra and draw what they would look like if they were unfolded	• HG4 (F96-S97) • HS1 -HS7 (S97-F98) • Shapes (F97) • JavaGami (F98)
ETS Surface Development	students match edges on folding nets to edges on line drawings of three-dimensional shapes	• HS1 -HS7 (S97-F98) • Shapes (F97) • JavaGami (F98)
ETS Cube Comparisons	students determine whether drawings of alphabet blocks could be drawings of the same block or must be drawings of different blocks	• HS1 -HS7 (S97-F98) • Shapes (F97) • JavaGami (F98)

Sections 5.2 through 5.7 present a discussion of assessment strategies used and the results obtained from different groups of students using HyperGami and JavaGami. A small group of students working only with paper shapes also participated in the assessment portion of the study. Section 5.8 attempts to create a picture of children's spatial learning by tying together the results obtained through the different assessment activities.

5.2 Net and Solid Card Matching

5.2.1 Description

The net and solid matching task was modeled after a study (Bourgeois, 1986) in which third graders were asked to match folding nets with their corresponding three-dimensional shapes. The goals of Bourgeois were:

> (a) to identify which common polyhedra are more easy to associate with their foldout shapes and which were more difficult; (b) to investigate whether different foldout representations of the same solid present different degrees of difficulty; and (c) to compare children's ability to make associates between foldout shapes and solids before and after a short-term experience with solids. (p. 222).

This study follows similar lines, but also seeks to investigate the strategies that children employ to arrive at their answers.

5.2.2 Method

Students in this portion of the assessment were given a set of paper polyhedra and then were shown 12 folding nets, one at a time. They were asked to pick out the polyhedron that would be formed by assembling the folding net on the card. Pictures of the folding nets used for this task are shown (reduced in size) in Figure 5-1 below. Pictures of the corresponding developed solid for each net are included for reference.

Each folding net formed a closed polyhedron; there were no incorrect nets. All of the corresponding solids for the nets were included among the group of paper polyhedra given to the students, but there were some polyhedra that did not match any of the nets given. Two of the polyhedra (the prism and pyramid) had two different folding nets among the set shown.

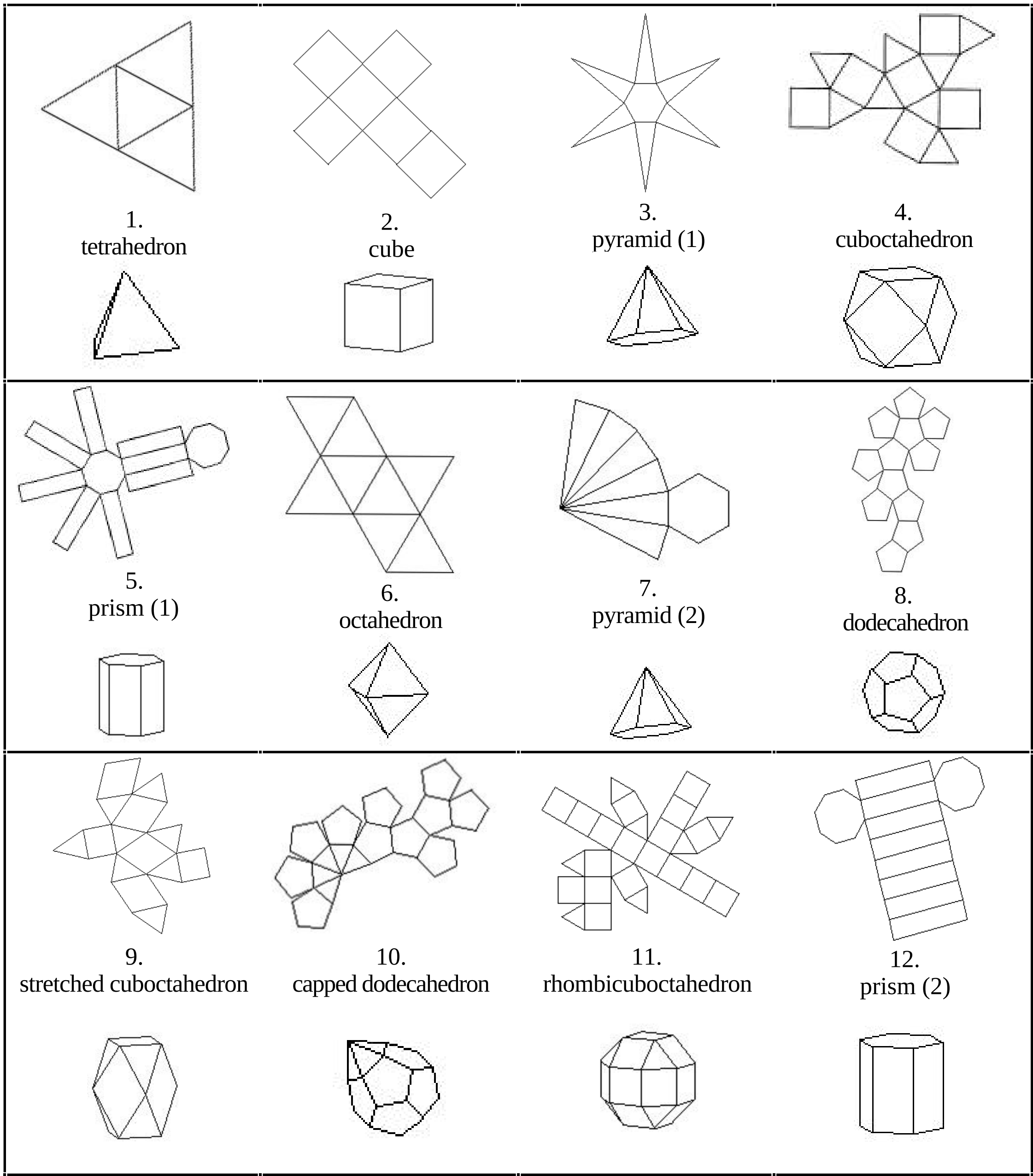


Figure 5-1. Folding nets used in net-solid matching task. The developed solid is also shown below each folding net.

5.2.3 Subjects

This was a small-scale pilot study to determine the kinds of tasks to use in assessment of children's thinking. Six HyperGami students in grades 3-6 (2 boys, 4 girls) took part in pre-tests, and four of the group completed post-tests after working with the software for approximately one academic semester.

5.2.4 Results

This task in general proved to be easy for most children to perform, both in the pre- and post-tests, and thus the results show a ceiling effect. Nonetheless, the task did lead to some insight into the techniques children used to mentally develop the solid object from its folding net.

Table 5-4 below shows the number of times each net was incorrectly matched with a solid object. These figures are counted from 9 trials: 5 pre-tests (one pair of sisters did the pre-test together, and this was counted as a single trial) and 4 post-tests. The two nets most often matched to solids incorrectly were the cuboctahedron (net number 4 in Figure 5-1) and the stretched cuboctahedron (net number 9 in Figure 5-1).

Table 5-4. Number of incorrect solid matchings (out of 5 pre-tests and 4 post-tests) for folding nets.

net	pre	post	net	pre	post
tetrahedron	0	0	pyramid 2	0	0
cube	0	0	dodecahedron	0	0
pyramid 1	0	0	stretched cuboct.	3	2
cuboctahedron	4	1	capped dodec.	0	1
prism 1	0	0	rhombicuboct.	0	0
octahedron	2	1	prism 2	0	0

- *Cuboctahedron and Rhombicuboctahedron*

In each of the five times that the net of the cuboctahedron was incorrectly matched, it was paired with the solid rhombicuboctahedron. The primary error that children made with this net is that they only looked for a shape with squares and triangles. However, in the post-test sessions, children were more aware of the configurations of the faces relative to one another. The cuboctahedron net was incorrectly paired with the rhombicuboctahedron in four of the five pre-tests given. The net was correctly matched with the solid in three of the four post-tests.

Student OCJ selected the rhombicuboctahedron for the cuboctahedron net in his pre-test, but in his post-test noticed that the rhombicuboctahedron had squares set together in a row, and thus could not match the net he was given.

> [OCJ, pre]: This. (He holds the rhombicuboctahedron) ... I kind of remembered it's like this -- one of those and then triangles (points to the net) ... because it has those and triangles and I was looking at the shape and I noticed that.
>
> [OCJ, post]: Yeeh. (He selects the cuboctahedron). This one -- it couldn't be like this one (he indicates the rhombicuboctahedron) *because these are like a row* ... squares and triangles.

Student NAB similarly selected the rhombicuboctahedron in her pre-test, but in her post-test correctly selected the cuboctahedron noting that the squares in the cuboctahedron were not next to one another:

> [NAB, pre]: ... This one (she holds the rhombicuboctahedron). You fold this one, and that one, and you also fold this one (points to faces on the net) ...
>
> [NAB, post]: (Pauses, then she picks the cuboctahedron). I think, wait, let me see again. (She compares the shape to the net). Yep, it's this one because it has the squares that way. *They're not by each other.*

- *Stretched Cuboctahedron*

The net of the stretched cuboctahedron was matched incorrectly to the (non-stretched) cuboctahedron in 5 of the 9 trials.

OCJ's strategy for selecting the correct shape was to methodically check for correspondences between the net and solid:

> [OCJ, post]: (Picks the stretched cuboctahedron). This one because (counts) ... these are these (points to the diamonds on the net and solid), these are these (does the same for the triangles), and these are these (indicates the squares).

The children who missed this shape and picked the cuboctahedron instead did not seem to recognize the slight difference between the side lengths of the squares and diamonds. Student EBH first selected the cuboctahedron, but then noticed the difference between the squares on the cuboctahedron and the diamonds on the cuboctahedron net:

> [EBH, pre]: (Selects the cuboctahedron). This goes over here, yeah ... Wait, ha! no -- that would be this (selects the stretched cuboctahedron) because of these big shapes -- the diamonds ... are only on this, so that's that.

- *Octahedron and icosahedron*

The octahedron net was incorrectly matched with the icosahedron in 3 of the 9 trials. The students who made this error looked for a shape with triangles, but did not count the triangles on the shape. Student NAB incorrectly selected the icosahedron in her pre-test, but in her post-test employed a new strategy:

> [NAB, post]: (She counts the triangles on the net). This one (she selects the octahedron). [How did you know that?] I counted them ... I didn't use that strategy ... last time I did it. [What made you think of using that strategy?] I wanted to do it because I think that's going to be helpful. And it is (giggles). [What did you do last time that was different?] I think I just looked at them ....

5.2.5 Discussion

With the exception of the cuboctahedron, stretched cuboctahedron, and octahedron, the nets on the test were very easy for children to match with corresponding solids. The cuboctahedron-rhombicuboctahedron and the octahedron-icosahedron combinations are instances of polyhedra which have the same types of faces but differ in numbers and arrangements of the faces relative to one another. Children who had the most success matching these solids looked not only at the shapes that made up the folding net, but also examined the relationship of the faces to one another, and took the time either to count the faces, or to make one-to-one correspondences between arrangements of faces on the nets and solid. The stretched cuboctahedron was confused with the (non-stretched) cuboctahedron when children did not notice that all but two of the quadrilaterals in the stretched case were parallelograms rather than squares and that all of the triangles were isosceles rather than equilateral.

Bourgeois (1983) found in his study with third graders that "... the difficulty of a net is a function of both the form of the solid and the arrangement of parts in the net" (p. 228). The children in the HyperGami study had no difficulty with the different configurations of polygons on the nets for the pyramid and prism, but some of these children were considerably older than Bourgeois' third graders. Perhaps more complicated nets with a larger number faces and more variations in the types of polygons would lend more insight into this idea.

Incorporating more complicated nets when working with children in grades 3-6 (or older) will provide more information about changes between pre- and post-test performances. Children's difficulties with the rhombicuboctahedron-cuboctahedron, and icosahedron-octahedron combinations indicate that it may be interesting to study children's strategies when working with nets with the same types of polygons, but with the polygons in different numbers and configurations. Other nets and solids composed of squares and equilateral triangles might be included with the cuboctahedron and rhombicuboctahedron -- these include the snub cube and square antiprism shown in Figure 5-2 (a) below. A regular pentagonal bipyramid (shown in Figure 5-2 (b)) might similarly be included with the icosahedron and octahedron. In the case of the stretched cuboctahedron-cuboctahedron error, it would be reasonable to include stretched counterparts of other shapes such as a stretched tetrahedron or stretched antiprism (Figure 5-2 (c)) to see if children make similar mistakes. It also may lend some insight to include nets and solids composed completely of parallelograms (such as the rhombic dodecahedron, shown in Figure 5-2 (d)) or irregular quadrilaterals (such as the dual of the rhombicuboctahedron shown in Figure 5-2 (e)).

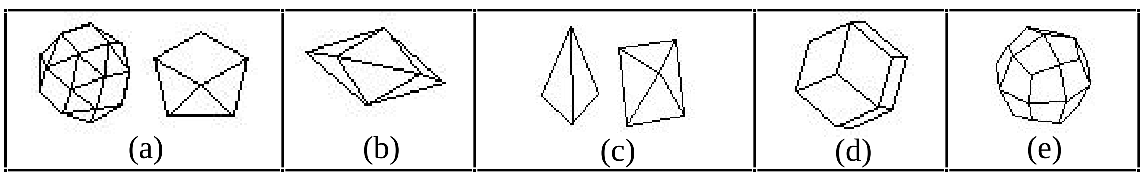


Figure 5-2. Polyhedra to add to the net-solid matching task. (a) snub cube and square antiprism; (b) regular pentagonal bipyramid; (c) stretched tetrahedron and stretched triangular antiprism; (d) rhombic dodecahedron; (e) dual of the rhombicuboctahedron.

5.3 Verbal Shape Description

5.3.1 Description

Students in this study were asked to describe paper polyhedra verbally. The purposes of the study were (1) to develop a way of categorizing children's shape descriptions; and (2) to determine whether any change takes place in children's verbal descriptions of three-dimensional shapes after working with JavaGami, HyperGami, or building paper shapes without software.

5.3.2 Method

Each student was given identical closed cardboard boxes, with each box containing a single polyhedron. The student was asked to take the polyhedron out of the box and describe it. The polyhedron itself was made from stiff cardstock paper and was either uncolored or tinted a neutral bluish-gray with black edge lines. For a similar study done with college undergraduates, cf. Eisenberg, Nishioka, and Schreiner (1997).

Table 5-5. Polyhedra used in the shape-description task.

Group	Polyhedra
HG3	dodecahedron, tetrahedron, icosahedron, cube, octahedron, capped dodecahedron, rhombicuboctahedron, cuboctahedron, truncated capped dodecahedron, stretched cuboctahedron, hexagonal pyramid, octagonal prism
HG4	cube, octagonal pyramid, cuboctahedron, octahedron, rhombic dodecahedron, tetrahedron
Paper Shapes	cube, octagonal pyramid, cuboctahedron, octahedron, rhombic dodecahedron, tetrahedron, capped cube, edge-capped cube
JavaGami	octahedron, stretched cuboctahedron, capped cube, truncated cube, sliced icosahedron, capped dodecahedron

5.3.3 Subjects

A preliminary version of this study was done with six HyperGami students (2 girls, 4 boys) in group HG2 in Spring 1995. This was not done as a pre- or post- test exercise, but simply to gain a very rough idea of the kinds of things that children would say about polyhedra and possible themes or trends to study further. One mistake made at this early stage was that children were given brightly decorated polyhedra which had been designed previously with the system, and the children consequently spent much time describing the decorations rather than the shapes themselves.

The test was then performed using undecorated shapes in the method described in 5.3.2 above. The same sets of shapes were included in pre- and post- tests. The tests were administered to groups HG3 (Fall 1995/Spring 1996), HG4 (Fall 1996/Spring 1997), Paper Shapes (Fall 1997) and JavaGami (Fall 1998). A total of 18 students (9 boys, 9 girls) completed both the pre- and post- tests; 5 other students (1 boy, 4 girls) took part in either the pre- or post- test but not both, usually due to logistical reasons.

5.3.4 Results

Analysis of children's descriptions points to the existence of certain categories of descriptions that occurred repeatedly:

- Visual characterizations: these are descriptions in which students describe a polyhedron only in terms of its visual appearance. These may include references to numbers of polygons, types of polygons in a shape, and numbers of vertices.
- Metaphorical descriptions: students relate the shapes to objects in the real world. These types of descriptions often begin with phrases such as: "this reminds me of ..." or "this looks like ...".
- Combinatorial descriptions: students describe polyhedra as combinations of other polyhedra -- "this is a pyramid on top of a cube" or "these are two pyramids put together".
- Functional descriptions: students describe polyhedra in terms of operations used to derive them from other, simpler shapes. These descriptions often include keywords such as "cap", "stretch", "slice", and "truncate" (operations that appear in the JavaGami system).
- Construction-based descriptions: students describe polyhedra in terms of real-world operations used to build them. They may use terms such as "gluing" or "folding."

A single description may also contain combinations of two or more different types of descriptions from the categories above. Table 5-6 contains representative examples of each type of description.

Table 5-6. Sample categorizations of children's polyhedron descriptions. Key parts of some descriptions have been italicized for emphasis.

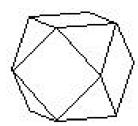
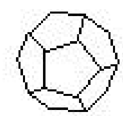
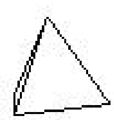
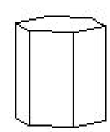
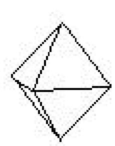
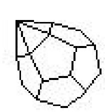
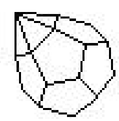
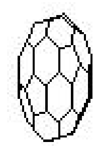
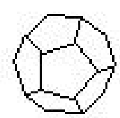
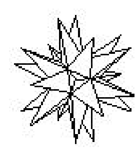
visual	[OCJ, describing cuboctahedron]  It's kind of like a pattern ... square, triangle, triangle; square triangle, triangle. That'd be like a geometric pattern -- does that on this side. Four, six, like a cube, kind of 'cept with triangles here ... (counts) like 14 sides.	[OJR, describing dodecahedron]  And this is a ... a ... what's the thing with 12 sides called? ... This has 12 pentagons and pentagons are ... two-dimensional shapes with five sides that are the same length and they have the same angles (that means they have the same lengths) ... and there's 12 of them put together to form a shape that -- has 12 of them. It's made of 12 pentagons.
metaphorical	[HSE, describing tetrahedron]  ... Many times I look at things in different ways. (tilts the shape) A fox ... ears, and ... or a little girl with a kerchief over her head. So I think of many things when I look at things like this. Sometimes I even think of a woodpecker (moves shape back and forth) ... sliding frogs ...	[LAA, describing prism]  This is more likely to be a trash can (drops shape on floor) Trash can. Or it could be a tin can, or a laundry basket, or it could be like a thing that you put on a ride ... the seats ...
combinatorial	[ARD, describing octahedron]  ... It's like two pyramids put together bottom to bottom ...	[NAA, describing capped dodecahedron]  ...it's round as this one (a dodecahedron) then it has ... I'll just use this one (she picks up tetrahedron, places it on one face of the dodecahedron). So it (the cap) would be like this except for five sides and much smaller.

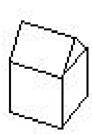
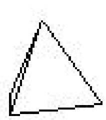
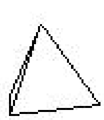
Table 5-6 (continued). Sample categorizations of children's polyhedron descriptions.

functional	[OCJ, capped dodecahedron]  ... It looks like a shape that someone *added on a five-sided pyramid thing ...*	[ULR, describing a stretched truncated icosahedron]  You can make circles or *stretched* circles -- just a little stretched so they're kind of like ovals, or you can make them really stretched so they're in the shape of a snake egg ...
construction-based	[AMO, describing dodecahedron]  This figure looks like a ball, but it also looks like strings of pentagons *either stapled or glued together.*	[ULR, describing a great stellated dodecahedron]  ... You make little triangular pieces - triangular-like pieces - a whole bunch of them. And then in the center you make a ... I think it's an icosahedron, *and you glue all the little triangular pieces to the icosahedron in the middle,* and when you're finished it looks like a star with lots and lots of points.

What follows is a discussion of some general conclusions drawn from the set of shape interviews collected from the children. Because of the danger of over-generalization from a small set of data, these ideas should be viewed as conjectures to be explored in larger-scale studies.

- *The post-test descriptions of children who only built paper shapes (without modeling them in software) became neither more procedural nor less metaphorical; however, when their descriptions changed, they were richer in both visual and construction-based language.*

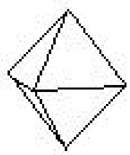
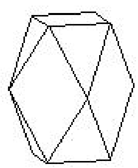
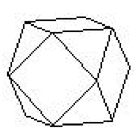
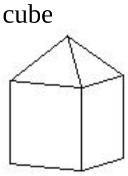
Table 5-7. Samples of pre- and post- shape descriptions by children who worked only with paper shapes. The classifications are shown by the bulleted points at the bottom of each box.

	Pre-test description	Post-test description
[LMM] describing edge-capped cube 	It's a -- real house! And it's a -- [turns it] Let's see, if you had a flat board that went across, it could be a teeter-totter [gestures]. It does look like the Children's museum just like the other one. Um, if I look at it from different angles it might look like something [...] Cheese cutter, except it's longer. • metaphorical	It's a another house. Um, it could be a piece of chalk, or a crayon, if you look at it upside down. Or, a broken pencil edge - pencil end. Um, it looks like a dog house. If there's like a little hole right there [points to one of the square faces] it could be a dog house ... It's a house. • metaphorical
[ELR] describing tetrahedron 	It's a triangle. [um hm, anything else?] And it has three sides. It has a very, very pointed tip. • visual	A duck. it's pyri-ad. Or is that? A pyri-ad. [giggles] [That's close, how would you describe it?] Triangles. [Okay, anything else] Any way you put it, it's still a triangle. But not like that [on a vertex] it's an upside-down triangle. • visual • metaphorical
[TSR] describing tetrahedron 	Oh, simple shape. Triangle. [how would you describe that?] Pyramid. This could be a building-block thingy-do, it could be a spinning thingy-do -- it doesn't spin very well. It could be one of those paper footballs the boys make... • metaphorical	It's your basic pyramid. That'd be very simple to do. *If you just took off these [faces] and put them down,* there'd be four points. Three points. Three points. This is the simplest shape that the Egyptians knew how to build. They really, really, liked this shape [not that simple when you are doing it in stone, though] The only shape they really built were pyramids. You'd think a square would be so much easier. [You mean a square pyramid?] Um, nah. It wouldn't be called pyramid. But a square would be easier to do because you wouldn't have to cut the stairs with an angle. [makes a triangular motion with her hands] So it would be a lot easier to do. • visual • construction-based

The examples in Table 5-7 above are representative descriptions from the children participating in the paper-shapes only study. In her pre-test, sixth-grade student LMM described the edge-capped cube in metaphorical terms: she spoke of it in terms of a house, teeter-totter, the Children's Museum in Denver, and a cheese cutter. Her post test was just as metaphorical, with comparisons to chalk, crayons, pencils, a dog house, and a house. ELR, a very young (second-grade) student, showed changes in her description of the tetrahedron. In her post-test description, she tried to name the shape, and she talked about the appearance of the shape from different angles: "Any way you put it it's still a triangle ..." TSR, a fifth-grade student who used many non-specific words such as "thingy-do" throughout her pre-test interview gave a much richer post-test description when she described the shape in terms of construction: "That'd be very simple to do. If you just took off these [faces] and put them down, there'd be four points." Her description then veered away from the subject and she described Egyptian pyramids and wondered why they did not built cubes ("squares") instead, but even this tangent is an improvement over her pre-test description: she has started to relate the geometry of shapes to her own definitions of how difficult or easy they are to build.

- *Children's descriptions became more procedural and less metaphorical after working with HyperGami or JavaGami.*
- *Their descriptions also incorporated more construction-based terminology.*

Table 5-8. Pre- and post- JavaGami shape descriptions by a seventh-grade girl. The classifications of her descriptions are shown by the bulleted points at the bottom of each box.

	Pre-test description	Post-test description
[APA] octahedron 	Okay. Well, it has eight sides and it's a diamond. And they're all little triangles and they're about the -- they're all the same size. It's made of paper. It has tons of lines on it. It has little, like, -- little pieces of brown stuff in the paper. I guess that's it. • visual	It's a diamond shape. It has -- it has eight triangles on it -- it could have been made by putting two pyramids together. You could have just taken a square and truncated it ... I guess that's pretty much it. It's symmetrical. • visual • combinatorial • functional
[APA] stretched cuboctahedron 	Oh, whoa. This one's weird. Okay. It has, well, it has four little diamond shapes, and ... eight little triangles and two squares. And it makes almost like -- it makes almost like a, um, [thinks] column. You know on like fancy lights, or whatever, there are those little designs on the top of them? ... • visual • metaphorical	Now, this one's annoying. I tried to find it on the computer and it turned out to be a simple little shape, stretched, when I was trying to truncate a rectangle. But, all you had to do is stretch it! It's got four diamonds, and eight triangles, and two squares -- probably pretty easy to fold. And it looks pretty cool, it could be like a jewelry box or something, if you left this part open ... • visual • functional • construction-based • metaphorical
[APA] cuboctahedron 	Oh, I've seen this shape before, somewhere, probably in here, but still ... It's got quite a few triangles, and quite a few of these shapes -- I'm not exactly sure what they are [octagons]... It looks almost like a ball of some kind -- it's like a ball with all straight sides -- I don't know, I guess -- a ball with all straight sides. • visual • metaphorical	Now this one's a square truncated. Yeah. Or it could have been just a shape, but -- it has eight little triangles, and I think these are hectagons? I don't know. Something like that. Laughs. And it has four hectagons ... It's probably just a shape, but you probably could have just truncated it, too. • visual • functional
[APA] capped cube 	Whoa. This one has five squares and four triangles, and it looks almost like a military fort, you know? It could have a door here. I don't know what else it could be. It could also be a lamp top [a what?] like on a fancy lamp. I'm not very good at this ... • visual • metaphorical	Aha. This one's a square with a cap on it. And, um, it's got five squares and four triangles, either that or you could have truncated a rectangle on the top of it to get this shape, but you probably just put a cap on it [mock accusingly]. • visual • combinatorial • functional

APA, a seventh-grader, described the shapes in considerably more informative terms after working with JavaGami. Her description of the octahedron changed from a purely visual one to a description that included both combinatorial -- "it could have been made by putting two pyramids together" and functional elements -- "You could have just taken a square (cube) and truncated it ..." Her visual and metaphorical initial description of the stretched cuboctahedron changed to one that included construction-based language: "...probably pretty easy to fold ...", as well as functional language: "...it turned out to be a simple little shape, stretched ..." In the post-interview, she described the other shapes in terms of "truncated" shapes and shapes with a "cap" on them -- her description of the capped cube in particular became much less metaphorical and more functional.

Table 5-9. Seventh grade boy's pre- and post- descriptions.

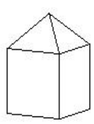
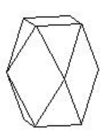
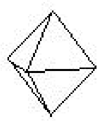
	Pre-test description	Post-test description
[ADH] capped cube 	I'd say this is a milk carton kind of thing, looks like a house. • metaphorical	I recognize all these shapes! Okay. This was my shape, two [...] This one was made by taking a cube and capping one side until it became a house. • functional
[ADH] stretched cuboctahedron 	I'd say the basic shape's a rectangle, or a hexagon, and then it's made three-dimensional by pulling out the sides ... I mean I guess that's all I can say about most of these. • functional	Oboy. I made this shape!! Okay, this shape was one of the ones I had to figure out how to do [theatrical "ahem"] with just the shape, so I can describe this very well. What I would do is, there's this really weird shape .. and it's just like this one chopped in half ... and it's under, I think, basic shapes, or something like that, and you just stretch it up until it becomes this shape. Yeah. Or you could make it by starting with a cube and stretching it and capping every single side... • functional

Table 5-9 (continued). Seventh grade boy's pre- and post- descriptions.

[ADH] octahedron 	I'd say it's like a three-dimensional diamond -- with bulges around the middle ... Like any angle, it's a three-dimensional diamond, no matter how you look at it. • metaphorical	Okay, well, this shape is sort-of a double-sided pyramid, like a 3d kite-shape I would say; and how would make it is make a pyramid, using a base of ... so there'd be a square and then four triangles on either side *so I could just fold them all up* and I would cap this end to make this [the pyramid] and I would cap it at probably two because it would make it equal to the other side. • metaphorical • functional • construction-based

Like APA, seventh grader ADH's post-test descriptions were markedly more function-oriented. He consistently described shapes in terms of operations and used words such as "capping", "chopped", and "stretch". He described the octahedron in terms of adding a cap to the base of the pyramid, "so I could just fold them all up and I would cap this end to make this ...".

5.3.5 Discussion

Ben-Chaim et al. (1989a) stress the importance of children employing a variety of representations of spatial information:

> Providing all pupils with the opportunity to explore a variety of types of representations of spatial and geometric information, as well as to communicate such representations should be a basic educational objective. (p. 121).

In a study of sixth to eighth grade students describing a three-dimensional structure using multiple representation modes (1989a), he reports that the students "dramatically improved their performance after three weeks of instruction in spatial visualization activities ... (which) did not specifically involve tasks similar to the Building Description Task." (p. 142).

The findings of this pilot study are in agreement with Ben-Chaim's conclusion. After work with HyperGami and JavaGami, students used fewer everyday metaphors and a larger number of functional and construction-based descriptions. Although metaphors -- like the one coined by a third-grader calling a capped dodecahedron a "drop of rain" -- are creative and invite visual playfulness, according to Triadafillidis (1995):

> Metaphors may provide a 'vivid and memorable' way of interpreting and describing knowledge (Lopez-Rial, 1989, 1990) ... This may have proved to be a disadvantage for the students since there are dangers of over-simplification and non-verified confusion between the different meanings that these metaphors have in everyday life and in the mathematics classroom. (p. 232).

The HyperGami and JavaGami participants appear to have a developed a richer set of resources to draw upon in their shape descriptions. It should be noted that work with HyperGami and JavaGami did not include any specific shape description lessons. The students who worked only with the folding nets of paper shapes (and did not model any shapes on the computer) showed increases in their use of real-world construction terms and more awareness of physical characteristics of the shapes, but they did not make dramatic moves away from everyday metaphor or increase their use of functional language. This suggests that the use of solid-modeling functions HyperGami and JavaGami provide a type of experience that does not occur when only assembling paper shapes. However, this is not to say that the real-world assembly of paper shapes is insignificant -- it is a valuable activity as well, since students become more aware of construction-based ways of describing shapes.

5.4 Polyhedron Drawings

5.4.1 Description

This study was an examination of students' drawings of polyhedra before and after working with HyperGami, JavaGami, or paper shapes alone. We did not expect the activities around the software or polyhedron construction to lead to differences between pre- and post-test drawings. Rather, this study was done in a spirit of exploration, to satisfy curiosity about this mode of shape representation and to see if any surprises might occur.

5.4.2 Method

Students were given polyhedra in closed identical cardboard boxes. They were asked to draw a picture of each polyhedron as it appeared three-dimensionally. Students were allowed to draw the polyhedron in any orientation they wished.

Table 5-10. Polyhedra used in the shape-drawing task.

Group(s)	Polyhedra used
HG4	cube, octagonal pyramid, cuboctahedron, octahedron, rhombic dodecahedron, tetrahedron
Paper Shapes JavaGami HS7	cube, octagonal pyramid, dodecahedron, octahedron (and tetrahedron for HS7)
HS1, HS2 HS4	cube, octagonal pyramid, cuboctahedron, octahedron, rhombic dodecahedron, tetrahedron
HS3, HS5 HS6	cube, octagonal pyramid, cuboctahedron, octahedron, rhombic dodecahedron, tetrahedron, capped cube, edge-capped cube

5.4.3 Subjects

This test was done with three students (2 boys, 1 girl) in group HG4, six students (2 boys, 4 girls) in the paper shapes only group, and seven students (4 boys, 3 girls) in the JavaGami group, for a total of 16 elementary and middle-school students. It was also done with all of the high school groups (HS1-HS7), for a total of 46 students (25 boys, 21 girls) in grades 9 through 12.

5.4.4 Results

As expected, there were no considerable differences in students' pre- and post-test drawings after working with HyperGami, JavaGami, or paper shapes alone. What did stand out upon examination of the results was the difficulty that students of all ages had in drawing the solids. Similar difficulties were exhibited by college undergraduates asked to draw some of the same polyhedra as the children in this study (cf. Eisenberg, Nishioka, and Schreiner, 1997).

5.4.5 Discussion

Mitchelmore (1978) developed an age-based classification scheme for children's drawings of regular solid figures. According to this scheme, drawings can be divided into roughly four stages (p. 230):

Stage 1 (preschematic - ages 4-7 years): Objects in this stage are represented by primitive schemata with circles and lines. A cube in this stage is drawn as a single square and a pyramid is represented by a single triangle.

Stage 2 (schematic - 7-9 years): Object features are drawn as if viewed orthogonally -- and a drawing at this stage "thus gives the impression of many viewpoints incorporated into one." (p. 230).

Stage 3 (prerealistic - 9-11 years): There is an attempt to draw the object from a single viewpoint in this stage; depth is shown through overlapping and size differences.

Stage 4 (realistic - 11+ years): Drawings in this stage show "visual realism" and successfully represent the object.

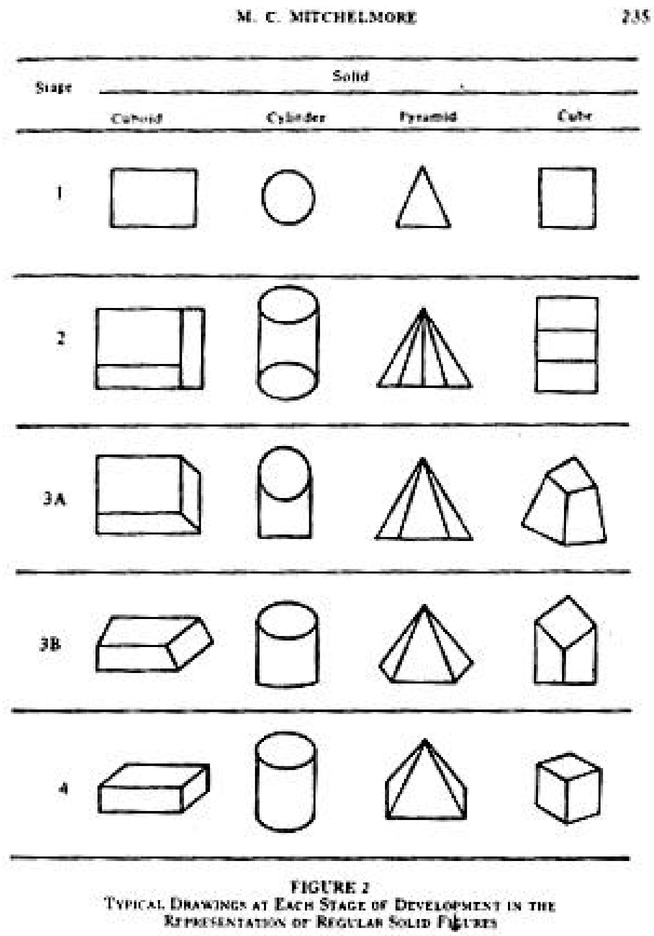
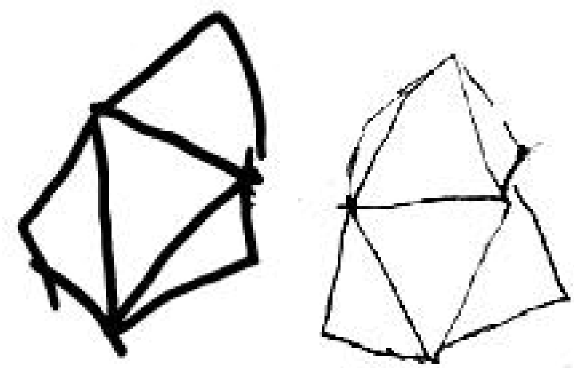


Figure 5-3. Mitchelmore's classification of drawings of regular solid figures. From Mitchelmore (1978), p. 235.

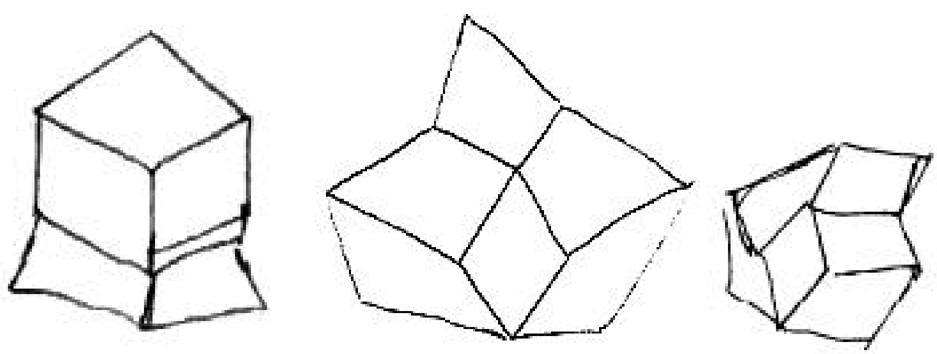
Some of the drawings by the high school students were representative of Mitchelmore's stage 2 and 3 drawings. The sketches of the octahedron in Figure 5-4 below show a representation from a single viewpoint (stage 3), but both of the drawings also show a "medley of viewpoints" (stage 2).



(a) (b)

Figure 5-4. Drawings of an octahedron by (a) NAT (9th grade boy) and (b) AMD (10th grade girl).

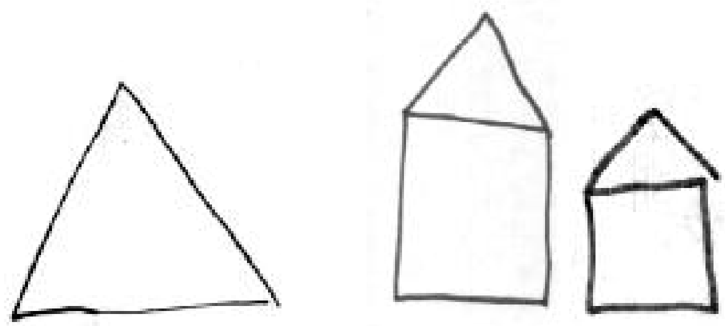
Students' drawings of the rhombic dodecahedron below also fall into Mitchelmore's stage 2 and 3 categorizations. Figure 5-5 (a) is a stage 2 representation with faces of the solid drawn orthogonally; Figures 5-5 (b) and (c) are stage 3 drawings with an attempt at perspective, but with some faces still drawn orthogonally.



(a) (b) (c)

Figure 5-5. Drawings of a rhombic dodecahedron by (a) ALD (12th grade girl); (b) RAP (9th grade boy); and (c) RAW (9th grade boy).

It is worth mentioning that a few students also produced stage 1 drawings -- such as the triangle representing the tetrahedron and the triangle-square combinations representing a capped cube in Figure 5-6 below. However, it is possible that the students were taking a lazy approach to drawing rather than experiencing genuine difficulty in representing the solids.



(a) (b) (c)

Figure 5-6. Drawings of (a) a tetrahedron by ELV (10th grade boy); (b) an edge-capped cube by BRH (9th grade girl); and the same edge-capped cube by (c) CAC (10th grade girl).

According to Mitchelmore, stage 2 drawings are done by children from 7 to 9 years old and stage 3 drawings come from children of 9 to 11 years old. The high school students' stage 2 and 3 drawings of polyhedra imply that these stages are not entirely dependent on age and might be more accurately characterized by the amount of experience students have had in representing shapes.

5.5 Folding Net Drawings

5.5.1 Description

Students were given paper polyhedra and were asked to draw what those solids would look like if they were unfolded and placed flat on the table. This exploration is based on Piaget and Inhelder's (1948) study of the rotation and development of surfaces in which children (5 to 10 years old) were asked to draw the unfolded versions of a cylinder, cone, cube, and pyramid.

The purposes of the study were (1) to develop a classification scheme specifically for folding nets of polyhedra; (2) to observe whether any consistent conceptual bugs recur in students' folding net drawings; and (3) to see if any changes in net representation take place as a result of students' work with HyperGami, JavaGami, or paper shapes alone.

5.5.2 Method

Students were given polyhedra in closed identical cardboard boxes. They were asked to draw a picture of the unfolded polyhedron. Students were allowed to handle the polyhedra as long as they did not physically take the shapes apart.

Table 5-11. Polyhedra given to student groups. Students were asked to draw what these shapes would look like unfolded.

Group	Polyhedra
HS1, HS2, HS4, HG4	cube, octagonal pyramid, cuboctahedron, octahedron, rhombic dodecahedron, tetrahedron
HS3, HS5, HS6, Paper Shapes	cube, octagonal pyramid, cuboctahedron, octahedron, rhombic dodecahedron, tetrahedron, capped cube, edge-capped cube
HS7, JavaGami	tetrahedron, cube, cuboctahedron, octagonal pyramid, capped cube

5.5.3 Subjects

This study was done as a pre- and post- test with all of the high school groups (HS1 to HS7), as well as groups HG4, JavaGami, and students working with paper shapes alone. A total of 46 high school students (25 boys, 21 girls) and 16 elementary and middle-school students (8 boys, 8 girls) took part in these activities.

5.5.4 Results

Students' nets spanned a wide range of representations. Tables 5-12 (a) and (b) on the two pages to follow offer a classification scheme along the lines of Mitchelmore's stages for solid drawings (Figure 5-4 above), but the system of net classification developed below is based on the characteristics of the polygons drawn and the relationships of the polygons to one another. It is not a series of age-related stages and with the exception of the correct nets, does not rank nets in terms of levels of sophistication.

A folding net might fit into more than one category using this classification scheme. For example, the orthogonal rendering of the cuboctahedron net might also fit under the 360-degree filled category, but it was classified according to the strongest feature of the drawing. For an alternative classification method of children's nets of everyday objects (including those with curved surfaces), see Potari and Spiliotopoulou (1992).

Table 5-12 (a). Categories of folding nets.

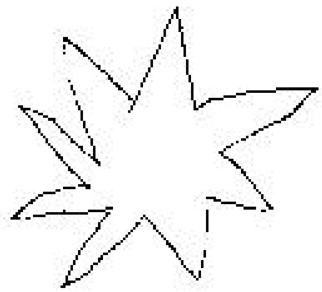
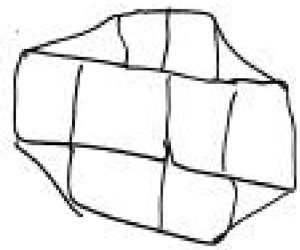
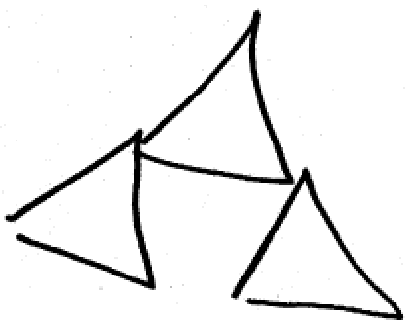
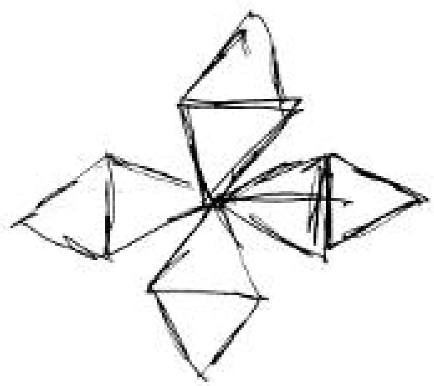
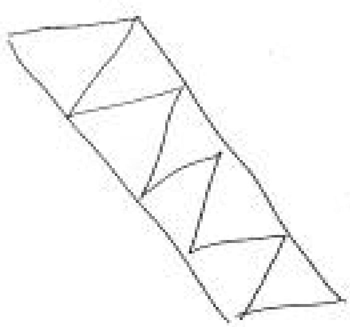
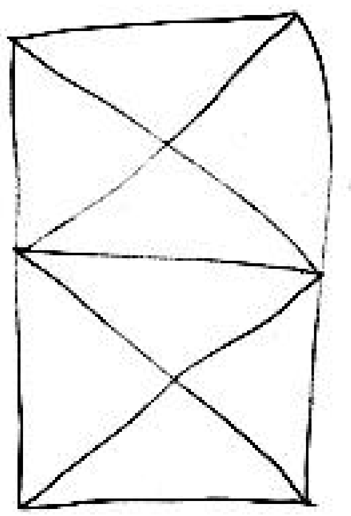
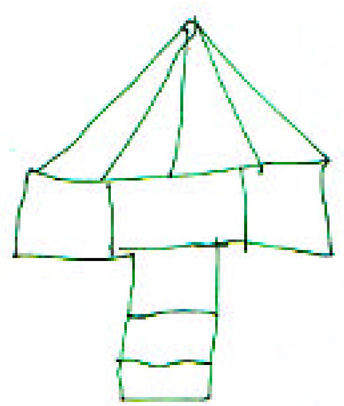
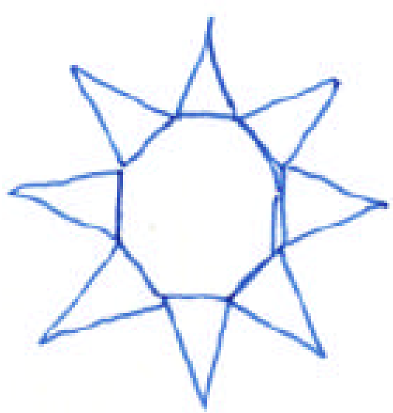
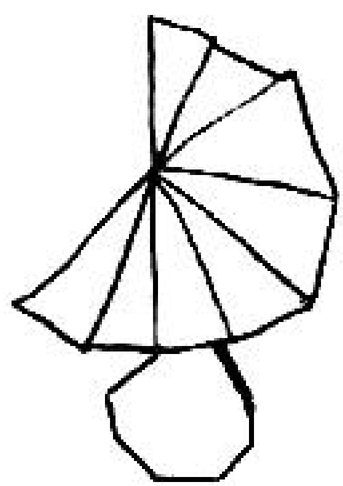
Unformed: the folding net is drawn as the outline of a shape without individual polygons.  octahedron by YDJ, grade 4	Orthogonal: polygons are drawn as projections from the solid onto the plane.  cuboctahedron net by HSG, grade 4
Disjoint: polygons are drawn disconnected from one another.  tetrahedron net by HSG, grade 4	Point-joined: polygons are joined at single vertices.  octahedron net by MIU, grade 9
Complete but incorrect: the net contains the correct number of polygons, but it does not fold into the solid.  octahedron net by IJW, grade 7	Correct but incomplete: the beginning of a correct folding net is drawn, but the net is missing polygons.  cuboctahedron net by CUS, grade 10

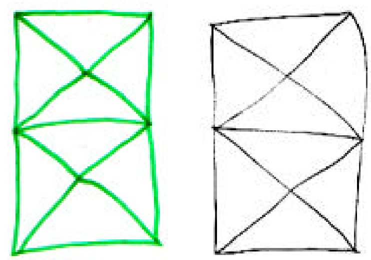
Table 5-12 (b). Categories of folding nets, continued.

360-degree filled: net polygons are drawn so that there is no angular deficiency between them.  octahedron net by TSR, grade 6	Edge error: the edge of one polygon is shorter than the edge of a tangent polygon.  capped cube net by APA, grade 7
Correct: the correct polygons of the net are connected at edges, and the net folds into the solid.  Octagonal pyramid by ALD, grade 12	Correct and crafted: the folding net is correct and created with an eye toward building the finished solid.  Octagonal pyramid by HCH, grade 4

5.5.5 Discussion

- *There were recurring patterns in the incorrectly drawn nets of specific polyhedra based on the geometry of the shape.*

In particular, the octahedron net (Figure 5-7 (a) and (b)) below was rendered as a 360-degree filled net by more than a few students -- the net appears to be composed of two sets of four triangles, but the triangles have no angular deficiency between them.



(a) (b)

Figure 5-7. Folding nets for the octahedron drawn by (a) MEB, grade 12 and (b)TSR, grade 6.

The cuboctahedron net (Table 5-13) was often drawn in an orthogonal fashion, with all of the polygons of the solid simply projected onto the plane of the net. This drawing strategy for the net spanned all age levels, implying that net drawing techniques are experience- rather than age-related.

Table 5-13. Orthogonally-drawn cuboctahedron nets.

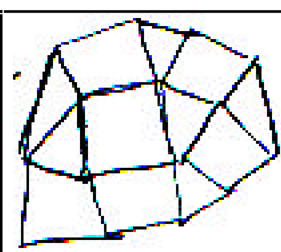
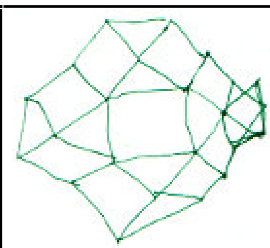
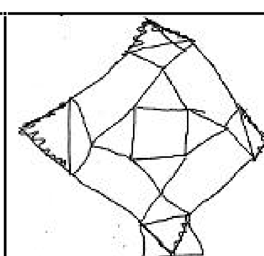
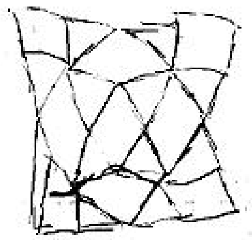
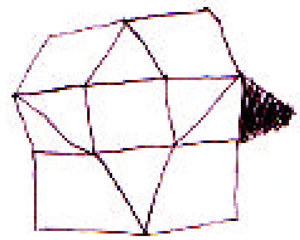
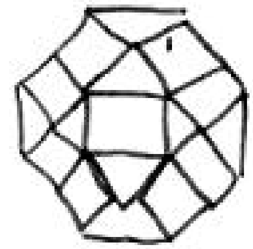
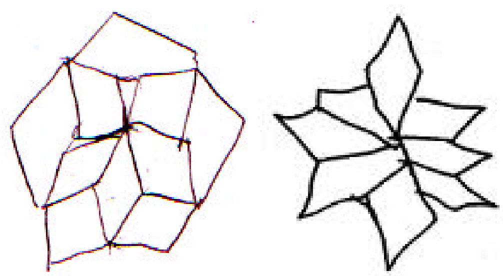
 GEL, grade 11	 VEA, grade 8	 IJW, grade 8

Table 5-13 (continued). Orthogonally-drawn cuboctahedron nets.

 KAL, grade 10	 HCH, grade 5	 IMX, grade 6

The rhombic dodecahedron net was similarly drawn in orthogonal fashion as shown in Figure 5-8. These patterns suggest a future study to investigate the relationship between the geometric characteristics of solids and students' strategies for unfolding them.



(a) (b)

Figure 5-8. Rhombic dodecahedron nets drawn by (a) LAP, grade 9; and (b) HSW, grade 4.

- *HyperGami and JavaGami students who initially drew unformed or orthogonal nets in their pre-tests drew a greater number of these nets correctly (or nearly correctly) in their post-tests.*

YDJ, a fourth-grade boy, drew the nets of the cube and the octahedron in an unformed fashion -- without differentiated polygons -- on his pre-test. After working with HyperGami for approximately two months, he drew a correct net for the cube. His folding net for the octahedron was still incorrect -- it has seven triangles rather than eight and five triangles surround a vertex with no angular deficiency -- but his post-test net is undeniably more sophisticated than his pre-test attempt.

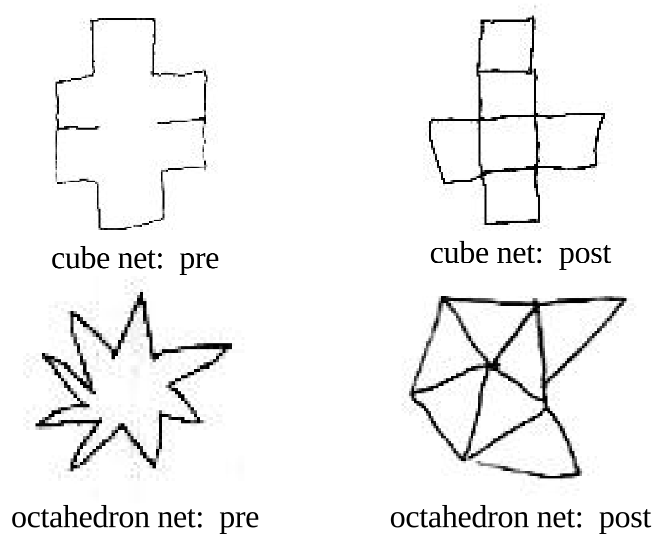


Figure 5-9. Pre- and post test nets drawn by a fourth-grade boy working with HyperGami.

High school students working with HyperGami also demonstrated improvement on post-test drawings. Student GEL, an eleventh-grade girl, represented the net of the capped cube by an incomplete and vertex-joined net on her pre-test (Figure 5-10 (a)). Her post-test net is complete and correct as shown in Figure 5-10 (b) below.

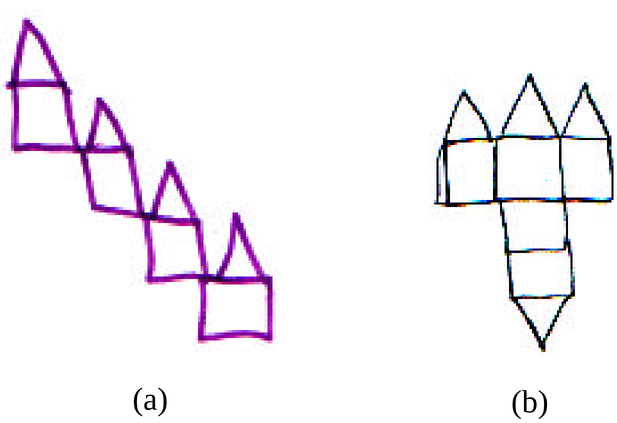
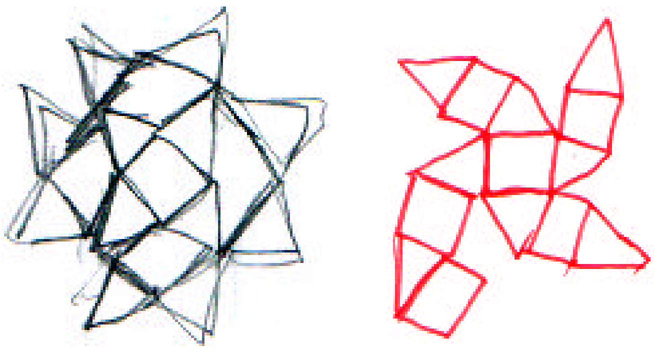


Figure 5-10. An eleventh-grade girl's (a) pre- and (b) post-test nets of a capped cube.

Ninth-grade student RIK drew an orthogonal net for the cuboctahedron in his pre-test, but designed a complete and correct net on his post-test:



(a) (b)

Figure 5-11. (a) Pre- and (b) post- nets of a cuboctahedron drawn by a ninth-grade boy.

Shifting focus from specific examples to the larger overall picture, Tables 5-14 to 5-17 summarize the types of folding nets drawn by elementary- and middle-school children on their pre- and post- tests. In the classification notation to follow, a "C" represents a correct net and "C" paired with any other letter represents a partially-correct net (that is, the folding net is valid, but there are some errors in the net). For the purposes of this analysis, a student received a "C" for any correct folding net, regardless of attention to craftsmanship as discussed in Table 5-11 and Table 5-12. Changes from a non-correct state to a correct or partially correct state are outlined in boldface. These tables are shown to provide the reader with an idea of the types of changes taking place in children's folding net drawings.

Table 5-14. Key to the symbols used in the folding net summary tables.

--	student did not attempt the problem	C	correct net
CN	the relationship of polygons to one another is generally correct, but incorrect number of polygons	CD	generally correct, but with minor errors in polygon shape/size
CX	the net contains the correct number and type of polygons, but it does not fold into the shape	CI	portion drawn is correct, but the net is incomplete
X	multiple errors	U	unformed
O	orthogonal	OC	partially orthogonal, but with some polygons drawn as a correct net
E	the edge of one polygon is shorter than the edge of a tangent polygon.	T	polygons are drawn without angular deficiency
D	disjoint polygons	V	polygons are joined at a single point

Table 5-15. Pre- and post- net classifications for elementary- and middle-school JavaGami students. Changes from incorrect to correct or partially-correct nets are outlined by bold rectangles; regressions from a correct net on the pre-test to an incorrect nets on the post-test are shaded.

	1	2	3	4	5
ACS - PRE	U	C	CX	C	C
ACS - POST	C	CI	CX	C	C
ADH-POST	CD	CI	O	C	C
ADH-POST	CD	C	CN	C	V
APA-PRE	C	C	O	C	CI
APA-POST	C	CD	--	CI	E
HCH-PRE	C	C	O	CD	C
HCH-POST	CD	C	OC	C	C
LAB-PRE	C	C	O	C	C
LAB-POST	C	C	CI	C	CX
EVA-PRE	C	C	O	OC	C
EVA-POST	C	C	O	C	C
AMS-PRE	O	O	O	CI	O
AMS-POST	C	C	CN	C	C

Table 5-16. Net classifications for elementary- and middle-school HyperGami students.

	1	2	3	4	5	6
IJW-PRE	C	C	O	O	O	C
IJW-POST	C	CN	C	CX	C	C
ADA-PRE	C	C	C	C	C	C
ADA-POST	C	C	C	C	C	C
YDJ-PRE	U	C	CX	U	O	CD
YDJ-POST	C	C	CX	O	O	CD

Table 5-17. Pre- and post- net classifications for paper shapes students -- students worked with either six or eight shapes, depending on their general mood by the end of the sixth shape on the pre-test. An "X" indicates that it was too difficult to classify the net because of numerous errors.

	1	2	3	4	5	6	7	8
HSG-PRE	CI	E	O	D	O	D	O	D
HSG-POST	CI	U	D	O	X	D	D	D
ELR-PRE	U	O	--	E	--	U		
ELR-POST	U	CD	O	O	O	LG		
EMM-PRE	C	CD	O	E	O	X	CN/D	X
EMM-POST	C	CD	O	O/T	O	C	C	X
IWH-PRE	C	X	CD	X	X	CD	C	X
IWH-POST	C	C	X	X	X	C	C	X
TSR-PRE	CI	T	--	T	--	C		
TSR-POST	C	T	--	T	--	C		
IMX-PRE	C	C	O	V	O/V	C	C	CD
IMX-POST	C	C	O	T	CX	C	C	CD

JavaGami and HyperGami children demonstrated noticeable gains in expertise in their drawings of folding nets. Children represented a larger number of nets in a correct or partially correct manner on the post-test compared to the orthogonal or unformed nets they had produced for the same shapes on the pre-test. Student AMS (a fifth-grade boy) showed the most striking changes in his net-drawing: he drew four of the five pre-test nets orthogonally, but after working with JavaGami, he produced four completely correct nets along with a fifth partially-correct net.

- *Students who worked with paper shapes but no software made modest gains in their net representations, but these changes were not as dramatic as the ones made by the students who worked with JavaGami and HyperGami.*[*]

Table 5-17 above summarizes the pre- and post- test results for the students who worked only with paper shapes and did not use either JavaGami or HyperGami. The results here are considerably less dramatic: there were proportionately fewer shifts from incorrect nets of any kind to correct or partially-correct ones.

The purpose of proposing a taxonomy of folding-net errors, as has been done in this section, is to lay a foundation for what might eventually be a finer-grained understanding of the types of errors that children (and adults) make in this task. The data presented in this thesis are still preliminary, but they do suggest that some types of errors are indeed more important -- further from correct -- than others. For instance, Table 5-15 reveals several students changing either from a CI (correct but incomplete) to C or vice-versa in their pre- and post-tests. It is at least reasonable to speculate, then, that a student drawing a CI net is experiencing fewer problems with the task than students producing orthogonal (O) nets; the only occurrences of O among post-tests are

[*] Note once again that "JavaGami" and "HyperGami" are references *both* to the software environments and the activities related to assembling the paper shapes generated with the software.

for those students whose pre-tests consisted of (what appear to be) more severe errors, such as O, D (disjoint), or U (unformed) nets.

5.6 Surface Development Test

5.6.1 Description

The Surface Development Test (Ekstrom et al. 1976a, Test VZ-3) was used as a test of near-transfer. In this test of mental paper-folding, the student was presented with a drawing of a two-dimensional folding net and was asked to determine which of the lettered edges on the solid match with the numbered edges of the net. Figure 5-12 below is a sample problem from the test:

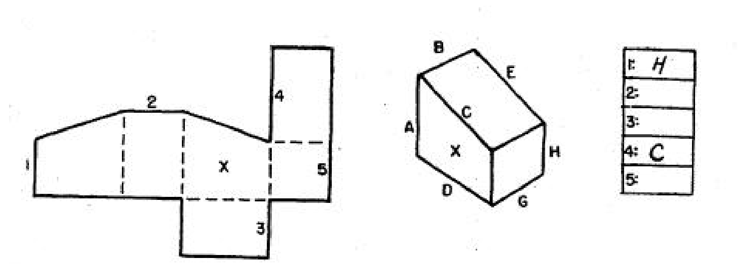


Figure 5-12. Sample question from the ETS Surface Development test (from Ekstrom, et al. 1976b, Test VZ-3).

5.6.2 Method

The test contained two 6-minute sections -- students were given the first section to do as a pre-test, and after 4-6 weeks of working with HyperGami were given the same section to do as a post-test. They were also given a second section during the post-test period to see if there were any learning effects from working through the first set of questions twice.

The figures on these tests were shapes that students would not see in the course of working with either HyperGami or JavaGami software, and thus the tests served as a measure of transfer.

5.6.3 Subjects

The test was administered to all of the high-school groups working with HyperGami. Although the test was designed for upper-level high school and college-age students, it was also done with the elementary- and middle-school students in the JavaGami and paper shapes groups.

5.6.4 Results and Discussion

Complete pre- and post- tests results were obtained from 37 high-school students and are summarized in Figure 5-13 below. Each tick mark on the x-axis represents a student; an "X" represents the student's pre-test score on part 1, and the square and diamond connected to the "X" represent the student's post-test score on parts 1 and 2, respectively.

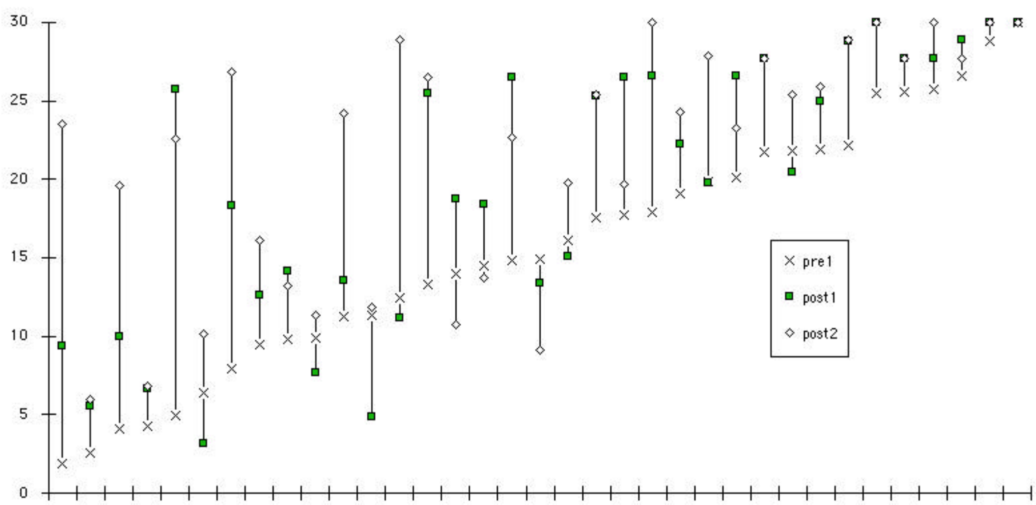


Figure 5-13. Pre- and post- test scores from 37 high school students on the ETS Surface Development test. Each mark on the x-axis represents a student. Shown on the chart are pre- and post-test scores for part 1, as well as scores for part 2 (administered only during the post-test).

Each section of the test consisted of six problems, with five possible points per problem. Each problem was scored as the number of correct answers (out of five) less a fraction of the number of incorrect answers. The average pre-test score on part 1 was

15.588 (SD=7.782), and the average post-test score was 19.527 (SD = 8.517). As shown in the figure, the majority of the students made gains in their scores. The average score on part 2 (given with the post-test) was 21.634 (SD = 7.609), again showing an increase over the pre-test scores. Approximately half of the high-school students taking this test increased their post-test scores by at least five points. A few of the students who scored in the bottom half of the sample on the pre-test improved their scores so that they ranked among the top-scorers in the class on their post-tests.

There was no clear pattern of increase or decrease between pre- and post-test scores among the JavaGami or paper shapes students, but it might be of interest to examine the scores of individual students as listed in Table 5-18 below. Although the elementary- and middle-school students scored considerably below the average of the high school students, sixth-grader IMX and seventh-grader ADH made considerable gains on their post-tests to place them among the higher-scoring high school students.

Table 5-18. ETS Surface Development Scores for JavaGami and Paper Shapes students. Two students in the Paper Shapes group declined to take the tests. Scores were calculated by subtracting a fraction of the number of incorrect answers from the number of correct answers. There were 30 possible points in each section.

JavaGami	grade level	pre1	post1	post2
LAB	5	1.905	0.762	4.256
HCH	5	4.752	2.929	12.833
ACS	7	7.124	10.233	15.917
AMS	5	8.037	4.545	7.327
APA	7	9.606	9.806	11.625
ADH	7	11.124	18.708	22.607
EVA	8	12.796	11.082	12.97
Paper Shapes				
HSG	5	1.054	-0.854	10.655
EMM	6	4.779	-1.139	3.833
IWH	6	8.445	12.749	15.262
IMX	6	12.713	23.092	22.048

5.7 Cube Rotations

5.7.1 Description

The ETS Cube Rotations test was used to measure students' ability to mentally rotate objects as a test of far-transfer. In this test, students were given two drawings of alphabet blocks and were asked to determine whether the diagrams *could* be drawings of the same block or whether the drawings must represent different blocks. The students were told that each face of a given block has a distinct symbol on it (there are no duplicate symbols on blocks) and it is impossible to tell what appears on the three hidden faces of a given block. Two sample questions appear below:

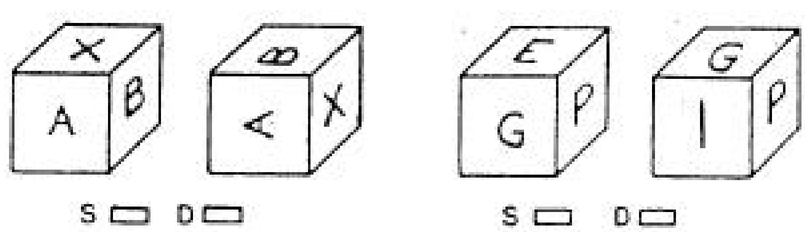


Figure 5-14. Sample questions from the ETS Cube Rotations test (from Ekstrom, et al. 1976b, Test S-2 Revised). The student is asked to mark whether the cubes could be the same ("S") or must be different ("D"), given that symbols do no repeat on a cube. In both cases, the correct answer is "D" indicating that the two cubes must be different.

5.7.2 Method

The first portion of the test (21 questions) was administered as a pre-test, and the same section was given as a post-test. A second part of the test (21 questions) was also given during the post-test period. Students had three minutes for each part of the test.

5.7.3 Subjects

Complete pre- and post-test results were obtained from 25 high-school students. There were fewer students who participated than in the cube comparisons sample partly because time constraints sometimes made it impossible to administer the test (it was always done last in a given battery of pre- or post- tests), and also because some students were too fatigued to take the test by the end of the period allocated to assessment. (This test was the portion of the assessment that by far met with the most grumbling from the high school participants).

This test was also administered to the younger JavaGami and paper shapes students, but these results will not be included in the study because these younger students sometimes misunderstood the directions (i.e., the distinction between "could be the same cube" and "must be different cubes") and stopped in the middle of the test to ask questions.

5.7.4 Results and Discussion

The average score out of a possible 21 points for the pre-test on part 1 was 5.833 (SD = 6.246) and the average for the post-test 9.083 (SD = 6.659). The average score on part 2 of the test administered only during the post-test period was 7.958 (SD = 7.339). The results for pre- and post-tests of part 1 are summarized in the chart below. Each tick mark on the x-axis represents a student; an "X" represents the pre-test score on part 1 for the student, and the square and diamond represent post-test scores on parts 1 and 2.

Approximately half of the students in the sample showed an increase in scores of at least five points (16.7%) on the post-test portion of part 1; the other half of the students experienced no change in scores, with a single student showing a decrease of more than five points on the post-test.

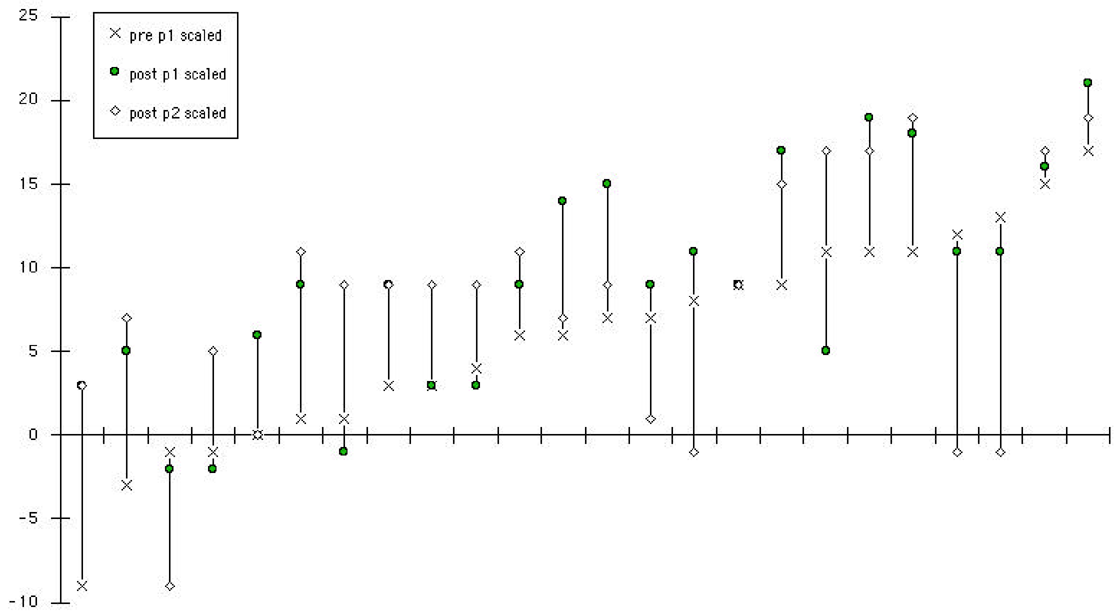


Figure 5-15. Pre- and post- test scores from 25 high school students on the ETS Cube Rotations test. Each mark on the x-axis represents a student. Shown on the chart are pre- and post-test scores for part 1, as well as scores for part 2 (administered only during the post-test). A few students had negative scores because scores were computed as the total number of items answered correctly less the number of items answered incorrectly.

5.8 Chapter Summary

This chapter has been an examination along a wide range of dimensions to draw together a picture of children's spatial learning as a result of building paper polyhedra or designing and building paper polyhedra with HyperGami and JavaGami software. A number of the pilot results are tantalizing: students have demonstrated richer functional descriptions after working with JavaGami and HyperGami; they have a more diverse set of conceptual tools for describing shapes after actually building them; interaction with the software (and to a lesser extent, work with paper shapes alone) has resulted in increased clarity and sophistication in their renderings of folding nets; and high school students from a wide range of academic backgrounds have shown solid improvement on standardized tests of visualization.

While these results are informative, they are not complete by any means. Certainly a larger number of subjects are needed (in particular among control groups, here represented solely by the small paper shapes group). The results of the shape description and net-drawing tasks indicate that there is an opportunity for a much finer-grained taxonomy of children's difficulties and strategies in these tasks. And finally, a more detailed analysis of the standardized ETS tests should be done to investigate the similarities in score increase rates between the Surface Development test and the Cube Rotations test: it is not clear from the results above whether the tasks are too closely related to measure the difference between near-transfer on the surface tasks and far-transfer on the rotations tasks.