1. Introduction

1.1 Overview

It is not surprising that spatial reasoning is heavily relied upon by artists and designers in their work. Davies and Talbot (1987) found that among 35 Royal Designers for Industry in the United Kingdom, they "often described a facility in using mental imagery ... as a multisensory and dynamic, mental sketch pad; using mental rotation and scanning images ...". (pp. 17-25). Artist Henry Moore uses a similar mental sketch pad when designing his sculptures:

> [He] gets the solid shape, as it were, inside his head ... He mentally visualizes a complex form from all round itself: he knows while he looks at one side what the other side is like ... (Ghiselin, 1952, p. 74).

What is perhaps less obvious is that the ability to think spatially plays an important part in scientific and mathematical problem solving. In a famous account of the way that Friedrich Kekulé determined the structure of the benzene ring during a dream, he recalls:

> ...again the atoms were gamboling before my eyes ... My mental eye ... could not distinguish larger structures ... all twining and twisting in snake-like motion. But look! What was that? One of the snakes had seized hold of its own tail and the form whirled mockingly before my eyes. As if by a flash of lightning I awoke. (Gardner, 1983, p. 191).

Many other scientists point to the essential role that spatial thinking plays in problem solving. Michael Faraday's visualization of lines of force around magnets and electric currents was instrumental in his formation of the concepts of magnetic and electric fields. (Roth, 1992). Albert Einstein said that "he rarely thought in words at all; his visual and 'muscular' images had to be translated 'laboriously' into conventional verbal and mathematical terms." (Ferguson, 1993, p. 45). Ludwig Boltzmann, Niels Bohr, and Werner Heisenberg also relied heavily on mental imagery as they developed modern physics. (Roth, 1992).

Even from his childhood, physicist Richard Feynman was aware of the role that visualization played in his thought processes. He shares a memory of a conversation with a friend:

> One time, we were discussing something -- we must have been eleven or twelve at the time -- and I said, "But thinking is nothing more than talking to yourself."
>
> "Oh yeah?" Bennie said. "Do you know the crazy shape of the crankshaft in a car?"
>
> "Yeah, what of it?"
>
> "Good. Now tell me: how did you describe it when you were talking to yourself?"
>
> So I learned from Bennie that thoughts can be visual as well as verbal.
>
> (Feynman, 1988, p. 54).

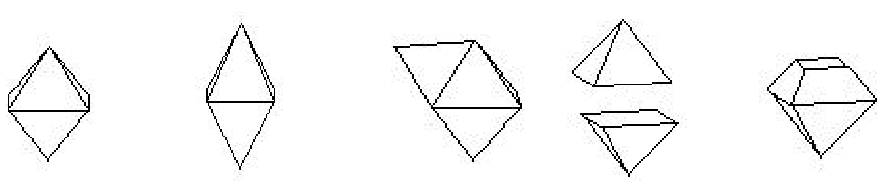
It is not difficult to find many quotes of this kind from famous scientists and mathematicians (cf. Grandin 1995, Gardner 1983). Of special interest to educators is that spatial ability is linked not only to the work of professional scientists, but it has been shown to be important in children's science and mathematics learning (cf. Siemankowski and Macknight, 1971; Guay and McDaniel, 1977; Poole and Stanley, 1972). Moreover, numerous studies with children have shown that spatial reasoning can be effectively taught, especially through the use of manipulatives. (cf. Brinkmann, 1966; Bishop, 1973; Mitchelmore 1980b; Ben-Chaim, 1988).

This thesis examines the design issues of combining real-world manipulatives and software to create an engaging environment in which children create mathematical paper sculpture and in the process gain practice in spatial thinking activities. The software environments created -- *HyperGami* and *JavaGami* -- are designed not to drill spatial concepts into students, but to provide a way for children to engage in activities that are at the same time enjoyable and spatially and mathematically rich.

1.2 Combining Manipulatives and Software

Mathematical manipulatives such as Polydron (M2), ZomeTool (M3), and Googolplex (M1) are examples of elegantly designed polyhedra-building systems. Using these manipulatives, children can assemble a wide variety of polyhedral forms by connecting pieces or snapping struts and joints together. Activities like these give children hands-on experience in building spatial structures and allow them to explore various relationships: how three-dimensional objects can be created by joining two-dimensional planes in the case of Polydron and Googolplex; or -- in the case of ZomeTool -- how different edge lengths and angles at which edges are joined will determine the geometry of the faces in a polyhedron. However, for all of their richness, the manipulative materials themselves are static objects, and the kinds of things that children can build are constrained by pre-determined shapes, sizes, and colors.

What software can provide is a way for students to *customize* polyhedra. In a software environment, a student would be able to see the effects of stretching a polyhedron along one of its axes; he would be able to add a pyramidal cap to a face of a polyhedron; he might slice a polyhedron into parts; or he may truncate a polyhedron at a single vertex.



(a) (b) (c) (d) (e)

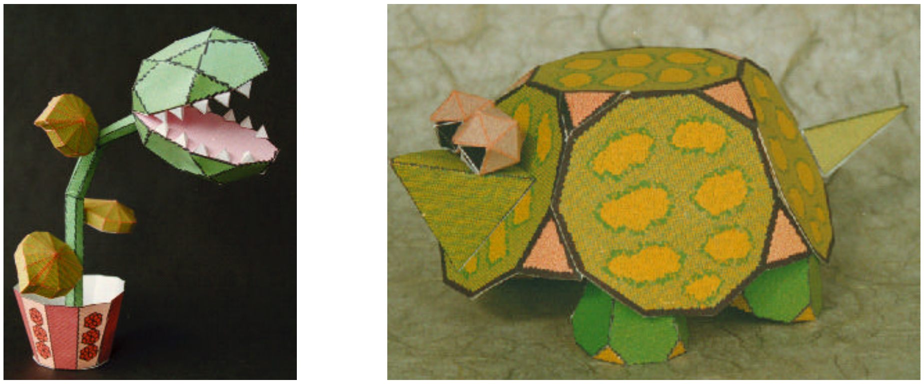
Figure 1-1. Operations on an octahedron. The octahedron in (a) is stretched along the z-axis in (b); has a pyramid added to one of its faces in (c); is sliced into parts in (d); and is truncated at a single vertex in (e).

If the software provides folding nets for solids, it will enable the student to then bring his activity back to the hands-on nature of manipulatives. He will work with real-world construction materials such as glue, scissors, and tape to assemble his folding nets into the polyhedra he has modeled on-screen. Once he is finished he has a set of manipulatives that he has customized, and as a next step, he can use these customized shapes to create his own multipart paper sculptures.

Along the way, the student has worked between many different two- and three-dimensional representations. The student has modeled the custom polyhedron by manipulating a two-dimensional representation of a solid object; decorated the folding net for the solid object on the screen; and assembled the two-dimensional folding net into a three-dimensional real-world model.

1.3 HyperGami

HyperGami (Eisenberg and Nishioka, 1997) is a programmable design environment for paper sculpture built by M. Eisenberg and the author as a tool for creating paper sculpture from polyhedra. Three-dimensional polyhedra are manipulated on the screen, and the system generates two-dimensional folding nets -- the flat patterns that fold into the polyhedra. These folding nets are decorated with a variety of on-line paint, pattern, and texture tools. The resulting net is printed by a color printer and is cut, glued, and assembled into a real paper model. The resulting polyhedral forms may then be combined to create multipart paper sculptures like the ones pictured in Figure 1-2 below.



(a) (b)

Figure 1-2. Mathematical paper sculpture created in HyperGami:
(a) Venus Flytrapohedron, and (b) Turtlehedron. Each polyhedron in a multipart sculpture is developed individually on-screen.

HyperGami is a Scheme-based system which contains a programming language interface where expressions are entered to perform functions (such as stretching, slicing, truncating, and capping) on polyhedra. Once a polyhedron has had a function applied to it, the new shape can then be operated upon in turn. This is HyperGami's central design principle: shapes and operations can be combined in a *shape algebra*, a way of creating an unlimited set of customized polyhedra.

HyperGami was pilot tested with elementary, middle-school, and high-school students over a period of four years. Students employed the system to create a diverse collection of polyhedral paper sculptures. Pre- and post- tests of students' spatial thinking evolved along with our work with children during this time period.

1.4 JavaGami

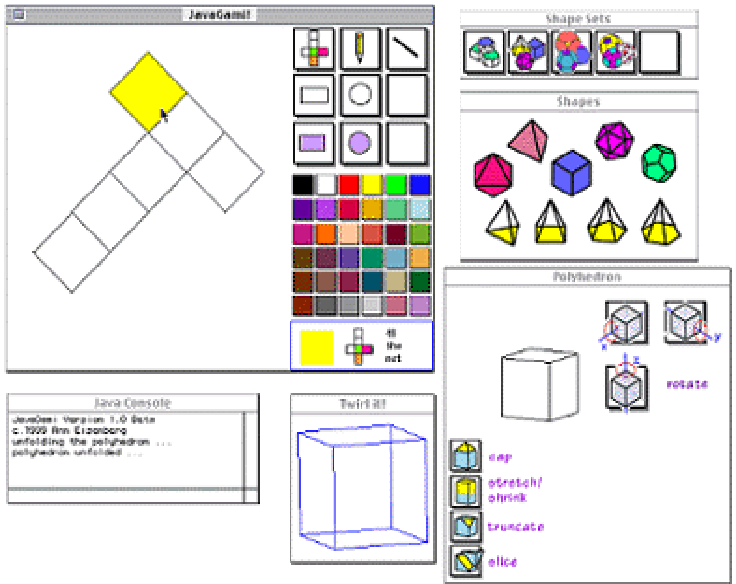


Figure 1-3. Overview of the JavaGami interface.

JavaGami is a software environment built by the author in response to students' feedback about HyperGami's user interface. JavaGami is built on the same paper-shape modeling principles as HyperGami, but interaction takes place only through direct manipulation (by moving the mouse and responding to simple dialog boxes) rather than through a combination of direct manipulation and entering programming language expressions. It has a small subset of HyperGami's shape modeling functions, but still supports real-time unfolding and provides tools for students to create a wide range of paper sculpture. It is more limited than HyperGami in terms of customizability of polyhedra, but it is also more accessible for a wide range of students, can be installed and run with relative ease, and is appropriate for use in a classroom setting. A pilot study of use of JavaGami as well as continued studies of children's spatial thinking has been completed with a small group of elementary and middle-school students.

1.5 Results

Changes in children's spatial thinking as a result of using HyperGami and JavaGami to create polyhedral paper sculpture are discussed in the results section of the thesis. A combination of assessment methods including children's drawings of two-dimensional folding nets, their verbal descriptions of polyhedra, and their performance on pre- and post- standardized tests of spatial visualization are discussed in an attempt to draw together a coherent picture of the effect of the software and polyhedron-building activities on their spatial reasoning. With a few caveats discussed in more detail in Chapter 5, students have shown increases in sophistication in their verbal descriptions of shapes, in their renderings of folding nets of shapes, and in their performance on standardized tests of spatial thinking after their work with JavaGami or HyperGami.

1.6 Reader's Guide

Chapter 2 is a discussion of spatial visualization, the correlation between spatial visualization ability and success in science and mathematics, a brief review of the trainability of spatial visualization using manipulatives, and the reasons for combining software environments with real-world manipulatives. Chapters 3 and 4 are descriptions of HyperGami and JavaGami, the two systems built for the design of polyhedral paper sculpture, and contain brief summaries of work with children on these systems. Chapter 5 is a discussion of children's spatial learning when using the software systems, and Chapter 6 is a sample case study of a child working with JavaGami. Chapter 7 is a summary of design lessons learned.