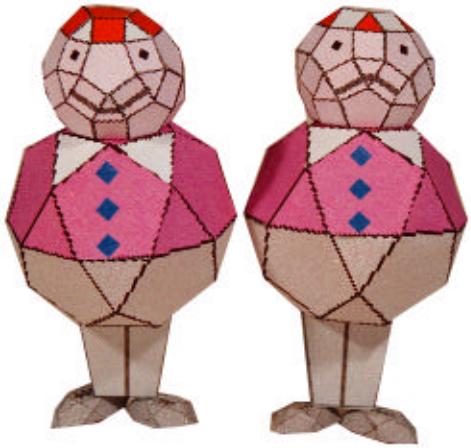
## Appendix A
## HyperGami Orihedra

*Orihedra* is a term we coined for mathematical paper sculpture. It is a combination of the words *ori*gami and poly*hedra*. Note that unlike origami, HyperGami orihedra are made by gluing tabs to edges, and the insides of the sculptures are hollow. All of the sculptures below were created by the author in collaboration with M. Eisenberg.

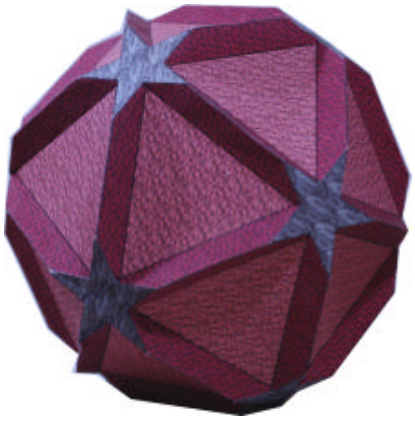
Pineapplehedron: each face of a stretched rhombicuboctahedron was covered with a pyramid. The leaves are trapezohedra truncated at one end.

A polyhedral pig playing a drum. The snout of the pig was made by capping and then truncating a dodecahedron. The body is a variant of one of the Archimedean duals.



Tweedle-Dee and Tweedle-Dum: the body of each sculpture is a icosidodecahedron, and the head is a small rhombicosidodecahedron. Note the use of decoration to create an impression of hats and clothing.

Finger Puppethedra: the heads of the wizard, king, and pirate are dodecahedra; the bodies are made from (left to right) a truncated pyramid, stretched cuboctahedron, antiprism, and prism.



A custom polyhedron: it is in the general shape of an icosahedron, with a star at each vertex.

Truncated icosahedron with a sliced cuboctahedron on each hexagonal face.

Gen, the Dragonhedron.

A Soma Cube puzzle rendered in paper.

Polyhedra may be rendered in many different media!

Rhombicuboctahedron with small adhesive-backed mirrors.

Rhombic dodecahedron made by pouring melted glycerin into a HyperGami-generated cardstock mold.

A small stellated dodecahedron printed on cardstock and then embossed with antiquing material.

Pillowhedra. The folding nets were generated and decorated in HyperGami and printed onto backprint material on a color printer. The nets were then heat- transferred to fabric and then were stitched and stuffed.

Candlehedron. Designs: turtle-graphics patterns printed onto tissue paper and then melted onto the candle. Candle: wax poured into cardstock truncated pyramid mold.

Icosahedron made by pouring plaster into a cardstock mold.

Icosahedron made by pouring melted chocolate into a cardstock mold. This was a favorite activity with the HyperGami students.

## Appendix B
## Work by Elementary and Middle-School Students in HyperGami

SuperDuck: A polyhedral duck with radioactive green prism feet by a fifth-grade boy.

A circus ride made from prisms and antiprisms by a third-grade girl. The people ride in the long antiprisms suspended from the horizontal axis.

A Christmas tree by a fifth-grade girl. She made the branches by slicing a pyramid and printing each branch level at a slightly smaller size than the level below it.

Danny: a sculpture design based on an eighth-grade girl's pet rooster. She attached real rooster feathers to his tail. The body of the rooster is a stretched rhombicuboctahedron; the head is a custom-capped cuboctahedron.

Polyhedral ornaments and gift-boxes designed by two fifth-grade girls.

Circus clown designed by a fifth-grade boy. The head of the clown is a great rhombicosidodecahedron and the tufts of hair are pyramids. The body, feet, arms, and hat are prisms.

Monster bug designed by a fourth-grade boy. The body of the bug is a truncated tetrahedron, the legs are prisms, and the head is an antiprism.

## Appendix C
## HyperGami Work by High School Students

Paper lantern in the shape of a great stellated dodecahedron. The student inserted a flashlight to illuminate it.

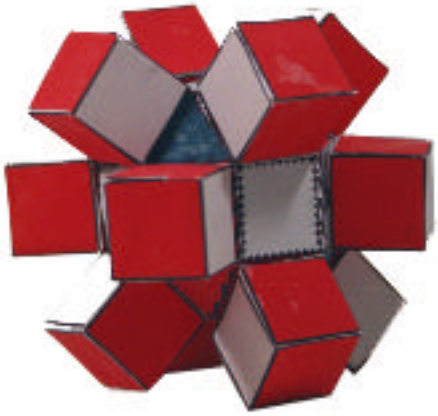
A dragon (left) and a flamingo (right). The dragon sculpture is composed of a number of Archimedean solids: the head is a truncated tetrahedron; the torso is a truncated dodecahedron; and the body is a truncated icosahedron. The head of the flamingo is a truncated octahedron.

Winnie-the-Pooh sculpture made from a dodecahedron for the head; a stretched dodecahedron for the body; and prisms for the arms, legs, and ears.

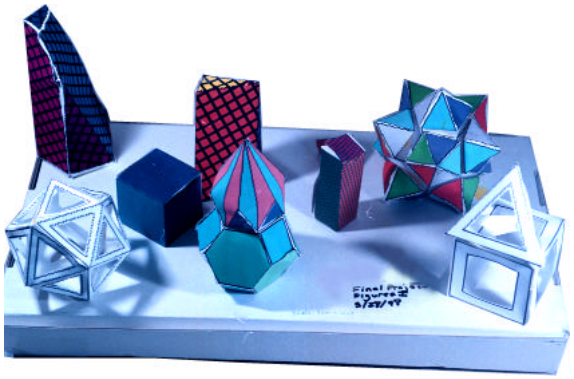
A student's pet cockatiel constructed from a truncated octahedron for the head; a stretched rhombicuboctahedron for the body; and custom prisms for the wings.

A paper bottle designed as a spoof on a popular ad.

Rhinoceros: the body is a stretched rhombicuboctahedron; the legs, horn, tail, and spikes are pyramids.



Cubes joined to a rhombicuboctahedron: a polyhedral sculpture based on a design by Alan Holden (1971).



A polyhedral city.

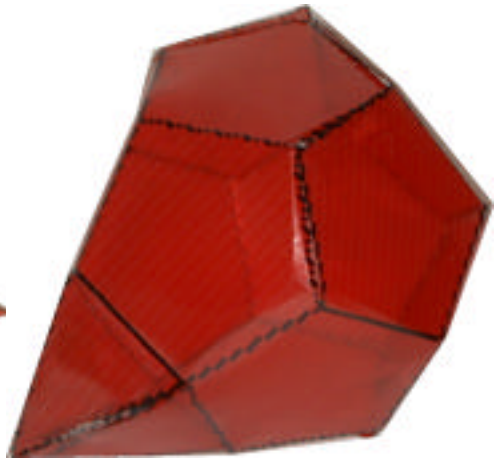
A kangaroo with a truncated tetrahedron body, pyramids for the head and ears; and prisms for legs and feet.

## Appendix D
## Work by Elementary and Middle-School Students in JavaGami

A stella-octangula made by a seventh-grade girl in JavaGami.

A capped pyramid made by a fourth-grade girl.

A star sculpture designed by a seventh-grade girl. She joined triangular pyramids to the central pentagonal pyramid piece. The outer pyramids touch at vertices in a rather unorthodox construction style.



A capped dodecahedron constructed from transparent material by a fifth-grade boy. He wanted it to look like a ruby.

## Appendix E
## Sample Assessment Procedures

### Net and Solid Matching

All of the solids listed below are placed on a table in front of the student. They are all approximately the same size. Each solid is numbered:

(1) dodecahedron
(2) tetrahedron
(3) icosahedron
(4) cube
(5) octahedron
(6) capped dodecahedron
(7) rhombicuboctahedron
(8) cuboctahedron
(9) capped dodecahedron truncated at one vertex
(10) cuboctahedron stretched 1.5 in z
(11) pyramid
(12) octagonal prism

The student is shown an example of a net folding into a solid.

The interviewer says:
"I am going to show you a picture like this one and ask you to pick the matching shape. There might be more than one picture for each shape. And some of the shapes might not have pictures."

The student is given a card with a folding net.

The student is asked:
"Can you show me which shape this is?"

The student is asked:
"How did you pick that shape?"

The procedure above is repeated for each card.

Points to note in this test:
1. Some of the solids will have more than one possible folding net.
2. Some of the solids will not have a corresponding folding net.
3. This test will NOT include "impossible nets".

Net/Solid types used:
1. Regular solids
2. Semi-regular solids
3. Solids from (1) and (2) above in which functions have been applied (truncation, mapping, capping, a combination of these)
4. 1 octagonal prism, 1 hexagonal pyramid (with two different nets generated)

Nets: same as (3) below with the following exceptions:
    1. include 2 nets for the rhombicuboctahedron

and none for the cuboctahedron
2. include the net for the icosidodecahedron
3. do not include nets for the capped truncated dodecahedron

**Solid Description/Solid Drawing/Net Drawing**

Protocol:

A. The week before the experiment:
reserve the videocamera
check stock of video-8 tapes
reserve the user study room

B. Preparations:
set up video camera
plug in microphone
turn off ringer on phone
assemble supplies -- boxes of shapes, answer forms, markers/crayons

C. The experiment:

Apparatus:

- Identical cardboard boxes to eliminate orientation bias when shapes are passed to the child.

- Paper polyhedra designed with HyperGami software, printed on an Apple Color Stylewriter 2400 on white cardstock. The polyhedra are colored in a neutral shade of blue.

Intro:
"I am studying the way that kids and adults think about shapes. We are going to do some activities with shapes made out of paper. I'm going to ask you to draw some things and to answer some questions. Remember that you can choose to not answer the questions if you do not want to. Please try your best, but don't worry that we are 'grading' you or anything -- we just want to see how you do."

Before the experiment:
Have the child pick a "code name" to write on his answer sheets.

Experiment:

1. Shape is passed to the child in a closed cardboard box to avoid orientation biases.
2. "Take the shape out of the box and look at it -- turn it around in your hands."
3. "If I wasn't in the room, and you wanted to describe what the shape looked like to me,
what would you tell me?"
4. Child is given a 3d-drawing answer sheet. "Now please draw the shape."
Retrieve the answer sheet.

5. Child is given a net-drawing answer sheet. "If we were to unfold the shape and put it flat onto the table, what would it look like? Please draw what this shape would look like if we unfolded it." Retrieve the answer sheet.
6. Child is given a net-discrimination sheet. "These are some pictures of shapes that have been unfolded. Circle the letter for all of the ones that can fold into the shape that you have. If none of them fold into your shape, circle the word "none". Retrieve the answer sheet, shape, and box.

At the end of the experiment:
Give the child a special treat.