CSCI 3104-Spring 2015: Assignment #10 (Paper and Pencil Portion)

Due date: Wednesday 4/29/2015, 4:30 PM (turn it in ECOT 724, CSCI 3104 mailbox)
Maximum Points: 40 points + 5 for legibility.

Note: This assignment must be turned in on paper, before class. Please do not email: it is very hard for us to keep track of email submissions. Further instructions are on the class page: http://csci3104.cs.colorado.edu

P1 (10 points) Suppose we wish to compute longest paths instead of shortest paths using Bellman Ford algorithm. Given a graph $G$, we modify it by negating all edge weights. Identify all graphs for which the modification of negating all edge weights does yield the longest path.

P2 (10 points) Use GLPSOL to solve the following LPs and write down the results.

\[
\begin{align*}
\text{max} & \quad 3x - 2y + z \\
\text{s.t.} & \quad x - y \leq 10 \\
& \quad y - z \leq 15 \\
& \quad x + 2y - 3z \leq 35 \\
& \quad x, y, z \geq 0 \\
\end{align*}
\]

\[
\begin{align*}
\text{min} & \quad x + y + z - w \\
\text{s.t.} & \quad x - y + z \leq 10 \\
& \quad x - 2y - z - w \leq 15 \\
& \quad x - 3w \leq 35 \\
& \quad w \leq 55 \\
& \quad x, y, z, w \geq 0 \\
\end{align*}
\]

P3 (20 points) The graph below shows a network of oil terminals for transporting oil from a source $s$ to sink $t$. Each edge $(i, j)$ in the network allows oil to flow from $i$ to $j$ in a unidirectional manner. It has a cost per unit $c_{i,j}$ and a capacity $W_{i,j}$ that limits the maximum units of oil that can be transported from $i$ to $j$. At any terminal other than the source or sink, the amount of oil flowing in through incoming edges should equal the amount of oil flowing out through outgoing edges. We wish to transport 100 units of oil from source $s$ to sink $t$ (i.e., 100 units must leave $s$ and enter $t$).

Write a linear program to find a minimum cost scheme that specifies how much oil should flow in each link and the overall cost of the scheme. Use GLPSOL/AMPL to solve your problem.
Hint: Use $x_{i,j}$ as a decision variable for the amount of oil flowing in edge from $i$ to $j$. The objective cost must be expressed in terms of $c_{i,j}$'s and $x_{i,j}$. Likewise, constraints must be added to ensure that incoming and outgoing flows match along with capacity constraints on the edges.