A Monotone Data Flow System for Analyzing Explicitly Parallel Programs

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Abstract

Reaching definitions information is vital for various code optimization algorithms. The problem of computing the reaching definitions information in sequential programs using a Monotone Data Flow System is well defined and understood. In this paper, we present Data Flow Equations to compute the reaching definitions information in explicitly parallel programs with post/wait synchronization. We also show that these equations form a Monotone Data Flow System.

1 Introduction

In [5], we have presented data flow equations to compute the reaching definitions information in explicitly parallel programs. In this paper, we prove that this data flow framework is a monotone data flow system (MDFS) using the definition of a MDFS as given by Kam and Ullman [6]. We first review the reaching definitions problem and the associated data flow system for sequential programs (section 2) and review the proof that it is an MDFS in §3. Similar proof techniques are used to prove that the data flow system in [5] is monotone. Before proving the monotonicity of the data flow system, we present the data flow equations in sections 4 and 5, followed by the actual proofs in section 6.

In the rest of this section, we point out the difference between analyzing sequential programs and parallel programs for the reaching definitions information.

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1.1 Reaching Definition Information in Sequential and Parallel Programs

Reaching definition information is the set of definitions reaching each use of a variable in a program. It is vital for various code optimizations; some of them include constant propagation, induction variable analysis, common subexpression elimination and dead code elimination.

In our work we consider the parallel extensions to FORTRAN as specified by the Parallel Computing Forum [8], which is the basis of the ANSI committee X3H5 standardization effort. The performance of parallel programs on existing and future high performance architectures depends to a great extent on the ability to perform aggressive code optimizations, including scalar optimizations across parallel constructs. Most of the existing compilers for parallel programs do not perform scalar optimizations across parallel constructs. Instead, they restrict optimizations to specific sequential sections of code in the parallel program.

Consider the sequential and parallel programs in Figure 1; these two programs have very similar control flow structures. The variable ‘j’ in 1(a) is not an induction variable, because the if . . . then may not be executed for each iteration of the loop. However, in the parallel program, ‘j’ is an induction variable since both branches of the Parallel Sections statement always execute for all iterations of the loop, but this could not be automatically detected without adequate dataflow information. Detecting such induction variables is useful for strength reduction, data dependence analysis and other optimizations. Likewise, dataflow information would show that the variable ‘k’ has the value 5 at the end of the parallel construct during each iteration.

2 Global Data Flow Analysis

The problem of global data flow analysis can be explained as follows [7]: given the control flow structure, we must discern the nature of the data flow (which definitions of program quantities can affect which uses) within the program. Data flow problems are often posed as a system of equations based on the Control Flow Graph of the program. A Control Flow Graph, CFG, of a program is a directed graph, \((V, E, V_0)\), where \(V\) is the set of vertices representing basic blocks in the program, \(E\) is the set of edges representing flow of control in the program and \(V_0\) is the unique node representing the entry into the program. We say a node \(P\) is the predecessor of node \(Q\) if there is an edge in the CFG from \(P\) to \(Q\). For each vertex in the CFG, we define
(1) \( j = 0 \)  
(1) \( k = 1 \)  
(2) loop  
(3) if (condition) then  
(4) \( j = j + 1 \)  
else  
(5) \( k = 5 \)  
(6) endif  
(6) \( l = k + 4 \)  
(7) endloop

(1) \( j = 0 \)  
(1) \( k = 1 \)  
(2) loop  
(3) Parallel Sections  
(3) Section A  
(4) \( j = j + 1 \)  
(5) Section B  
(5) \( k = 5 \)  
(6) End Parallel Sections  
(6) \( l = k + 4 \)  
(7) endloop

(a) Sequential Program  
(b) Parallel Program

Figure 1: Example sequential and parallel programs.

![Control Flow Graph]

Figure 2: Control Flow Graph for the sequential program in Figure 1.
some \textit{basic} attributes, which can be defined unambiguously from an analysis of the program. Then we define \textit{inherited} and \textit{synthesized} attributes in a set of data flow equations, and solve these equations.

This section discusses the data flow equations to solve the \textbf{reaching definitions} problem in sequential programs \cite{1}. The reaching definition problem is to find the set of definitions of a variable \texttt{v} that can reach a particular \texttt{use} of \texttt{v}. This is also referred to as the \textit{ud-chaining} problem in the literature. In the later sections of the paper, we explain how these equations can be extended to solve the \textbf{reaching definitions} problem across parallel constructs in explicitly parallel programs.

\subsection{Reaching Definitions}

We say a definition of a variable \texttt{v} \textit{reaches} a point \texttt{p} in the program if there is a path in the CFG from that definition to \texttt{p}, such that no other definitions of \texttt{v} appear on the path. To determine the definitions that can reach a given point in a program, we first assign a distinct label to each definition. Our problem is to be able to find for each node \texttt{n} of the CFG, \textit{In (n)}, the set of definitions that reach the beginning of \texttt{n}.

Formally, a definition \texttt{d} of a variable name \texttt{v} reaches a node \texttt{n} if there is a path \texttt{n}_1, \texttt{n}_2, \ldots, \texttt{n}_k, \texttt{n} in the flow graph such that

1. \texttt{d} is within \texttt{n}_1,
2. \texttt{d} is not subsequently killed in \texttt{n}_1 (i.e., \texttt{v} is not redefined) and
3. \texttt{d} is not killed in any of \texttt{n}_2, \ldots, \texttt{n}_k.

One way of calculating \textit{In (n)} is to determine all generated definitions and then to propagate each definition from the point of generation to \texttt{n}. An easy way of doing this is to solve the following set of 2N simultaneous equations for a CFG of N nodes:

\[
Out(n) = (In(n) - Kill(n)) \cup Gen(n)
\]

\[
In(n) = \bigcup_{p \in \text{pred}(n)} Out(p).
\]
<table>
<thead>
<tr>
<th>Node</th>
<th>Gen</th>
<th>Kill</th>
<th>In</th>
<th>Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry</td>
<td>{}</td>
<td>{}</td>
<td>{}</td>
<td>{}</td>
</tr>
<tr>
<td>(1)</td>
<td>$j_1$, $k_1$</td>
<td>$j_4$, $k_5$</td>
<td>{}</td>
<td>$j_1$, $k_1$</td>
</tr>
<tr>
<td>(2)</td>
<td>{}</td>
<td>{}</td>
<td>$j_1$, $k_1$</td>
<td>$j_1$, $k_1$</td>
</tr>
<tr>
<td>(3)</td>
<td>{}</td>
<td>{}</td>
<td>$j_1$, $k_1$</td>
<td>$j_1$, $k_1$</td>
</tr>
<tr>
<td>(4)</td>
<td>$j_4$</td>
<td>$j_1$</td>
<td>$j_1$, $k_1$</td>
<td>$j_4$, $k_1$</td>
</tr>
<tr>
<td>(5)</td>
<td>$k_5$</td>
<td>$k_1$</td>
<td>$j_1$, $k_1$</td>
<td>$j_1$, $k_6$</td>
</tr>
<tr>
<td>(6)</td>
<td>$l_6$</td>
<td>{}</td>
<td>$j_1$, $j_1$, $j_4$, $k_5$</td>
<td>$l_6$, $j_1$, $k_1$, $j_4$, $k_5$</td>
</tr>
<tr>
<td>(7)</td>
<td>{}</td>
<td>{}</td>
<td>$j_1$, $k_1$</td>
<td>$j_1$, $k_1$</td>
</tr>
<tr>
<td>Exit</td>
<td>{}</td>
<td>{}</td>
<td>$j_1$, $k_1$</td>
<td>$j_1$, $k_1$</td>
</tr>
</tbody>
</table>

| Entry | {} | {} | {} | {} |
| (1) | $j_1$, $k_1$ | $j_4$, $k_5$ | {} | $j_1$, $k_1$ |
| (2) | {} | {} | $j_1$, $j_1$, $j_4$, $k_5$ | $l_6$, $j_1$, $k_1$, $j_4$, $k_5$ |
| (3) | {} | {} | $l_6$, $j_1$, $j_1$, $j_4$, $k_5$ | $l_6$, $j_1$, $j_4$, $k_5$ |
| (4) | $j_4$ | $j_1$ | $l_6$, $j_1$, $j_1$, $j_4$, $k_5$ | $l_6$, $j_4$, $k_1$, $k_5$ |
| (5) | $k_5$ | $k_1$ | $l_6$, $j_1$, $k_1$, $j_4$, $k_5$ | $l_6$, $j_1$, $j_4$, $k_5$ |
| (6) | $l_6$ | {} | $j_1$, $j_1$, $j_4$, $k_5$ | $l_6$, $j_1$, $k_1$, $j_4$, $k_5$ |
| (7) | {} | {} | $j_1$, $k_1$, $j_4$, $k_5$ | $l_6$, $j_1$, $k_1$, $j_4$, $k_5$ |
| Exit | {} | {} | $j_1$, $k_1$, $j_4$, $k_5$ | $l_6$, $j_1$, $k_1$, $j_4$, $k_5$ |

**Table 1:** Table showing two iterations of the data flow equations to solve the reaching definitions problem for the sequential program in Figure 1(a).
Out (n) is similar to In (n) but pertains to the point immediately after the end of the basic block. Kill (n) is the set of definitions outside of n that define variables that also have definitions within n and Gen (n) is the set of definitions generated within n that reach the end of n. We are interested in the smallest solution possible for In, which is why we start with In as the empty set for all n. The algorithm that computes the In sets starts with this initial approximation and iterates through the above set of equations until a fixpoint is reached. This particular set of equations and the iterative algorithm form a monotone dataflow system. In such a system, the order of traversal of the CFG only affects the convergence rate of the different sets to their fixpoint. It has been proven that a depth first traversal of the CFG helps reduce the number of iterations to five in most practical cases [1].

The CFG for the sequential program in Figure 1 is given in Figure 2. Variable ‘j’ is defined at nodes (1) and (4); call these j₁ and j₄ respectively. The reaching definitions for the use of ‘j’ at node (6) are j₁ and j₄. The In, Out, Kill and Gen sets for the different nodes are given in Table 1. This table shows two iterations of the data flow equations; the third iteration is the same as the second, indicating that a fixpoint has been reached.

This example illustrates how the In and Out sets are computed for sequential programs, given the Gen and Kill sets. In the next section we prove that the data flow framework for the reaching definitions problem presented in this section forms an MDFS.

3 Monotone Data Flow Framework

Definition 1 A Monotone data flow analysis framework is a triple \( D = (L, \cap, F) \), where

1. \( L \) is a bounded semilattice with meet \( \wedge \).

2. \( F \) is a monotone function space associated with \( L \).

Definition 2: Given a bounded semilattice \( L \), a set of functions \( F \) on \( L \) is said to be a monotone function space associated with \( L \) if the following conditions are satisfied:

M1. Each \( f \in F \) satisfies the monotonicity condition,

\[ \forall x, y \in L, \forall f \in F, x \leq y \Rightarrow f(x) \leq f(y) \]
M2. There exists an identity function e in F, such that
\[ \forall x \in L, e(x) = x \]

M3. F is closed under function composition, i.e., \( f, g \in F \Rightarrow fg \in F \), where
\[ \forall x \in L, fg(x) = f(g(x)) \]

M4. For each \( x \in L \), there exists an \( f \in F \) such that \( x = f(\perp) \).

In the rest of this section, we show that the data flow framework to solve the reaching definitions problem in sequential programs (given in section 2) is a monotone data flow system. The DFS for this problem is \( RDEF = (L, \cap, F) \), where \( L \) is a lattice whose elements are in the power set of the set of definitions in the program (a definition can be represented as the pair, ( variable, flow graph node )). The meet operator on this lattice is set intersection and \( \leq \) is set inclusion. The function space \( F \) consists of functions of the form:
\[ f_i(x) = (x - K_i) + G_{i} \]

where, \( K_i \) and \( G_{i} \) are the Kill set and Gen set for a specific node in the Control Flow Graph of the program. This function is referred to as a transfer function, where the variable \( x \) corresponds to the In set of our data flow equations and \( f_i(x) \) corresponds to the Out set.

**Theorem 1** \( RDEF = (L, \wedge, F) \) is a monotone data flow system.

**Proof:** Clearly, \( L \) is a semi-lattice with \( \perp \) equal to the empty set and \( \top \) equal to the set of all definitions in the program. To prove that \( F \) is a monotone function space, we prove that each of the four conditions given in Definition 2 are satisfied by \( F \).

**Proof of M1:** \( \forall x, y \in L, \text{if } x \leq y \text{ then, } x \subseteq y \). Since \( K_i \) and \( G_{i} \) are constants for a given function, \( (x - K_i) + G_{i} \) is a subset of \( (y - K_i) + G_{i} \), i.e., \( f(x) \subseteq f(y) \).

**Proof of M2:** There is an identity function \( e \), such that, \( e(x) = x \) for all \( x \in L \), defined as follows:
\[ e(x) = (x - K) + G \]

where, \( K = G = \emptyset \)
Proof of M3: Using bit vector notation, each transfer function has essentially two characteristics, the Pres set (i.e. the complement of the Kill set) and the Gen set. Given all the $2^n$ possible sets ($n$ is the number of bits or definitions), we have $2^n \times 2^n$ possible transfer functions, one for each combination of Pres and Gen sets. Each basic block must use one of these transfer functions. The composition of any two transfer functions $f_i$ and $f_j$ in $F$ gives us:

$$f_i(f_j(x)) = (f_j(x) \cap Gen_i) \cup Pres_i$$

i.e.,

$$f_i(f_j(x)) = (x \cap (Pres_i \cap Pres_j)) \cup (Gen_i \cup (Pres_i \cap Gen_j))$$

So, if $Gen_{ij} = (Gen_i \cup (Pres_i \cap Gen_j))$ and $Pres_{ij} = Pres_i \cap Pres_j$, then, we have,

$$f_i(f_j(x)) = (x \cap Pres_{ij}) \cup Gen_{ij} = f_if_j(x)$$

Since $F$ contains all possible transfer functions, $f_if_j \in F$. □

Proof of M4: For any given $x \in L$, we can always find a transfer function $f \in F$ such that

$$f(y) = x, \forall y \in L.$$ This is possible if we choose $Gen = x$ and $Pres = \emptyset$. □

Since $L$ is a semi-lattice with a meet operator and $F$ is a monotone function space, RDEF $= (L, \cap, F)$ is a monotone data flow system.

□

4 Parallel Constructs and the Parallel Flow Graph

In this paper, we only consider the Parallel Sections construct [8]. The Parallel Sections construct is used to specify parallel execution of explicitly identified sections of code. Each section of code is interpreted as a parallel thread, and must be data independent except where an appropriate synchronization mechanism is used. The Parallel Sections construct can also be nested, appear in the body of a loop and so on.

We consider synchronization between threads in the form of event synchronization, described by a binary event variable. Operations are available to indicate that an event has occurred (post), to ensure that an event has occurred (wait), and to indicate that an event has not occurred (clear). In our work, we only consider post and wait statements.

When a post statement is executed, the appropriate shared variables are made consistent and the value of the event is set to "posted", no matter what its value was previously. When
(Entry) event( ev )
(Entry) x = 2
(Entry) y = 5
(1) loop
(2) Parallel Sections
(3) Section A
(3) if (condition) then
(4) x = 7
(4) post( ev )
else
(5) x = 8
(5) post( ev )
(6) endif
(6) z = y * 7
(7) Section B
(7) Parallel Sections
(8) Section B1
(8) wait( ev )
(8) x = x * 32
(9) Section B2
(9) z = y * 54
(10) End Parallel Sections
(11) End Parallel Sections
(11) y = x * z
(12) endloop

Figure 3: Parallel Program with Parallel Sections and event synchronization
a \texttt{wait} statement is executed, the appropriate shared variables are made consistent and the thread waits for the event to be marked "posted".

An example parallel program with \texttt{Parallel Sections} construct and event synchronization is shown in Figure 3. Section A and B execute in parallel. Within section B, there is a nested \texttt{Parallel Sections} construct where sections B1 and B2 can execute in parallel. The event variable \texttt{ev} will be posted in one of the branches of the if-construct, depending on the value of \texttt{condition}. The execution of Section B1 can not proceed until at least one of the post occurs. Note that the \texttt{Parallel Sections} is inside a loop. This example is purely illustrative; in particular, the event variable \texttt{ev} is not cleared between iterations of the loop, and thus, this example would not execute properly. We refer to this example in §6, to show the interaction of loops and synchronization variables. Note that this is a \textit{sequential} loop; analysis of parallel loops is a topic of future papers.

The language standard does not define the memory consistency model for the target architecture. Rather, it allows a range of implementations including copy-in/copy-out semantics. We assume copy-in/copy-out semantics in the compiler, because it provides more opportunity for optimization. For example, within a single thread, we are free to load copies of variable values into registers or propagate subexpressions and the like, disregarding the actions of other threads. This does not imply that we implement a pure copy-in/copy-out program. Rather, we use this as one model of memory consistency because it is convenient for compiler optimizations and allowed by the language standard. Correct programs should obey copy-in/copy-out semantics as well as other memory consistency models allowed by the language standard.

At a \texttt{fork} point, i.e., a \texttt{Parallel Sections} statement, every branch of the fork (each thread) gets its own copy of the shared variables. Each thread modifies its own local copy and at the \texttt{join} point, i.e., the \texttt{End Parallel Sections} statement, the copies from the different threads are merged with the global values. In the presence of post/wait synchronization, the thread that waits for an event to occur updates its copy with the values from all the threads posting that event. Multiple copies of a variable may \textit{potentially} reach a wait statement, either because of multiple posts executed by different threads or because of one or more posts (executed by different threads) and the \texttt{waiting} thread defines that variable prior to the \texttt{wait} statement. Some decision has to be made at run time as to which value will reach the \texttt{wait} statement. However, at the compiler level, we allow more than one value to reach that point and the
Figure 4: Parallel Flow Graph for the example parallel program.

presence of multiple values at such wait statements indicates potential anomalies.\footnote{Similarly at a join node, multiple values for a variable reaching that node indicates a potential anomaly in the Parallel Sections construct.} Similarly at a join node, multiple values for a variable reaching that node indicates a potential anomaly in the Parallel Sections construct.

4.1 Parallel Flow Graph

In this section, we describe the Parallel Flow Graph, a data structure used to represent control flow, parallelism and synchronization in explicitly parallel programs. The Parallel Flow Graph (PFG) is similar to the Synchronized Control Flow Graph \cite{4} and the Program Execution Graph \cite{2}. A PFG is basically a directed graph with nodes representing extended basic blocks in the program and edges representing either sequential control flow, parallel control flow or

\footnote{Note that multiple values reaching a wait statement do not necessarily mean there are anomalous updates; for example, the post statements may have been conditionally executed.}

11
synchronization. An extended basic block is a basic block that may have at most one wait statement at the start of the basic block and at most one post or branch statement at the end of the basic block. A sequential control flow edge represents sequential flow of control within sequential parts of the program. A parallel control flow edge represents parallel control flow, as at fork and join points in the program. Finally, a synchronization edge is an edge from a post statement to a corresponding wait statement.

The PFG for the parallel program in Figure 3 is shown in Figure 4. Nodes (2) and (7) represent fork nodes and nodes (11) and (10) are the respective join nodes. Sequential, parallel and synchronization edges are identified in this figure as indicated.

5 Data Flow Equations for Parallel Sections

In Section 2, we reviewed the data flow equations from [1] to compute the reaching definition information at any point in a sequential program. In this Section, we extend these equations to handle the Parallel Sections construct. The extensions are based on the following fundamental concepts:

- At parallel branch points, such as fork nodes, all the branches execute; in the case of sequential branch points, e.g., if-statements, only one of the branches will be executed.

- A value defined at a point prior to a parallel construct does not reach the corresponding parallel merge point if it is always killed in at least one of the branches. In contrast, for sequential branches, the value would need to be always killed along every branch.

- The compiler must assume that a conditionally defined value in a parallel section may reach the parallel merge point. These definitions do not kill the definitions prior to the Parallel Sections statement. In actuality, only one definition reaches the merge point, but determining the actual reaching definition is undecidable. Thus, the compiler must be conservative and assume that both definitions reach.

These concepts are illustrated by the sequential and parallel programs in Figure 5 and by the program in Figure 6 on page 13. The values of the variable ‘a’ reaching the sequential and parallel merge points (i.e., the endif and End Parallel Sections statement respectively) in Figure 5 are different. In the case of the sequential program, the values of the variable ‘a’ reaching the endif statement is either the value defined before the if test or the value defined in the then-part of the if-construct. However, at the parallel merge point, the only reaching value of ‘a’ is the value defined in Section A. In Figure 6, the variable ‘c’ is defined conditionally
(1) \( a = 0 \)  
(1) \( b = 1 \)  
(2) if (condition) then  
(2) Parallel Sections  
(3) \( a = a + 1 \)  
(3) Section A  
(3) \( b = 7 \)  
(3) \( b = 7 \)  
else  
(4) \( b = 5 \)  
(4) Section B  
(4) endif  
(4) End Parallel Sections  
(5) \( c = a \ast b \)  
(5) \( c = a \ast b \)  

(A) Example Sequential Program  

(B) Example Parallel Program

Figure 5: Example sequential and parallel programs.

(1) \( a = 0 \)  
(1) \( b = 1 \)  
(1) \( c = 2 \)  
(2) Parallel Sections  
(3) Section A  
(3) \( a = a + 1 \)  
(3) \( b = 7 \)  
(4) Section B  
(4) Parallel Sections  
(5) Section B1  
(5) \( b = 5 \)  
(6) Section B2  
(6) if (P) then  
(7) \( c = 6 \)  
(8) endif  
(9) End Parallel Sections  
(10) End Parallel Sections  
(10) \( d = a \ast b + c \)

Figure 6: Example parallel program to illustrate data flow equations.
in Section B. Therefore, this value and the value of ‘c’ defined prior to the outer Parallel Sections construct reach the parallel merge points. The sequential data flow equations (Section 2) will not be able to handle such cases. The new data flow equations for parallel programs must still be able to say that the values of ‘b’ in Figure 5 reaching the join node are either from Section A or Section B. As mentioned earlier, more than one value of a variable reaching a parallel merge point indicates a potential anomaly in the program.

We introduce two new sets to the the ACCKillin and ACCKillout sets data flow framework of Section 2. These sets accumulate definitions that occur outside a parallel construct and that are killed along specific parallel branches in the parallel construct. The ACCKillin set at a node is the set propagated by its predecessors and ACCKillout set at the node is its ACCKillin set updated by the definitions killed in this node, excluding the definitions generated in this node, i.e., its Kill set minus the Gen set. In our first example (Figure 5), the accumulated kill set at the end of Section A is the value of ‘a’ defined prior to the parallel construct because the definition of ‘a’ inside Section A will always kill the previous definition.

Parallel sections can be nested, but the information represented by the ACCKillout set pertains to a single parallel block. For example, in Figure 5, the ACCKillin set at the entry to the parallel program is empty. At node (1), the ACCKillout set includes ‘a3’ since it is in its Kill set. However, if we propagate this set via Section B, that does not define ‘a’, to the parallel merge node, the ACCKill set at this node will contain this definition. However, ‘a3’ always reaches the parallel merge point and should not be in the ACCKill set of any of its parallel predecessors. Therefore, we clear ACCKillout at fork nodes and use this empty set in computing the accumulated kill sets inside the corresponding parallel block. We must also preserve the current value across internal nested parallel blocks because a join node must have access to the ACCKillout set from the corresponding fork node. Thus, fork nodes store the ACCKillout, computed from its Gen and Kill sets in another set, ForkKill, and a ‘technical edge’ between corresponding fork and join nodes makes this information available to the join node. At join nodes, the In set will exclude definitions from the ACCKillout sets of all the parallel predecessors of this node.

We propagate the ACCKill sets by computing the ACCKillin set at a merge node as the union of the ACCKillout sets of its parallel predecessors and the intersection of the ACCKillout sets of its sequential predecessors.
In sequential programs, we define \textit{Kill} \((n)\) to be the set of all the definitions of variables outside \(n\) for those variables defined in \(n\); these are the definitions that will be overridden when the variable is defined in node \(n\). This is appropriate for sequential programs or a single thread of control because assignments can not occur in parallel.

By comparison, in the case of parallel programs, where we can have multiple simultaneous threads of execution, we distinguish between the \textit{Kill} set and the \textit{ParallelKill} set. The \textit{Kill} set for node \(n\) contains all killed definitions from nodes that can not execute at the same time as node \(n\). Similarly, the \textit{ParallelKill} for \(n\) contains all definitions from nodes that can execute at the same time. For example, in Figure 5(B), the \textit{Kill} set of section \(B\) contains the definition \('b_1'\) (the definition of \(b\) from node 1), while the \textit{ParallelKill} set contains the definition \('b_2'\).

We would expect both definitions \('b_3'\) and \('b_4'\), but not \('b_1'\), to reach the join node (node 5). Definition \('b_1'\) should not reach because there are assignments to \('b'\) that are guaranteed to occur later in the execution order. Both \('b_3'\) and \('b_4'\) should reach the join node because the compiler can not assume a particular execution order or memory semantics. Indeed, this indicates a potential data anomaly or race condition in this particular program. We segregate the kill sets into \textit{Kill} and \textit{ParallelKill} sets to distinguish between these cases. \textit{ParallelKill} \((n)\) can be computed by traversing the PFG and including those definitions \(d_i\) of variables \('v'\) such that \('v'\) has a definition in \(n\) and \(d_i\) occurs in a node that \textit{can} execute in parallel with \(n\). This can be done by traversing the parallel flow edges and the sequential flow edges in all \textit{Sections} that have the same fork node and join node as the \textit{Section} \(S_n\) corresponding to \(n\) but not \(S_n\) itself. Thus, as in the sequential data flow problem, \textit{Kill} and \textit{ParallelKill} can be computed directly and need not be computed using an iterative algorithm.

The \textit{ACCKill} sets accumulate information about definitions that are killed \textit{within} a sequential thread, and we include the \textit{Kill} sets in the \textit{ACCKillin} and \textit{ACCKillout} sets. We do not include the \textit{ParallelKill} set because that set represents information about other threads where the temporal ordering of definitions is undefined. When computing the \textit{Out} set for each node, we must consider all killed definitions, i.e. the union of the \textit{Kill} and \textit{ParallelKill} sets.

The data flow equations for the reaching definitions problem in programs that have the \textit{Parallel Sections} construct is given in Figure 7. In those equations, \textit{par.pred} refers to the set of parallel flow predecessors of the node; \textit{seq.pred} refers to the set of sequential flow predecessors of the node and \textit{pred} refers to the set of all predecessors (both parallel and sequential flow
\[ \text{Out}(n) = \text{In}(n) - \text{Kill}(n) - \text{ParallelKill}(n) \cup \text{Gen}(n) \]
\[ \text{In}(n) = \bigcup_{p \in \text{pred}(n)} \text{Out}(p) - \bigcup_{p \in \text{par_pred}(n)} \text{ACCKillout}(p) \]

\[ \text{ACCKillout}(n) = \begin{cases} 
\emptyset & (n \text{ is a fork node}) \\
(\text{ACCKillin}(n) + \text{Kill}(n)) - \text{Gen}(n) & (n \text{ is a join node, with corresponding fork node } f) \\
+ (\text{ForkKill}(f) - \text{Out}(n)) & \\
(\text{ACCKillin}(n) + \text{Kill}(n)) - \text{Gen}(n) & (\text{otherwise})
\end{cases} \]

\[ \text{ACCKillin}(n) = \bigcup_{p \in \text{par_pred}(n)} \text{ACCKillout}(p) + \bigcap_{p \in \text{seq_pred}(n)} \text{ACCKillout}(p) \]

\[ \text{ForkKill}(n) = \begin{cases} 
(\text{ACCKillin}(n) + \text{Kill}(n)) - \text{Gen}(n) & (n \text{ is a fork node}) \\
\emptyset & (\text{otherwise})
\end{cases} \]

**Figure 7:** Dataflow Equations for Programs with Parallel Sections
<table>
<thead>
<tr>
<th>Node</th>
<th>Gen</th>
<th>Kill</th>
<th>ParKill</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>${a_1, b_1, c_1}$</td>
<td>${a_3, b_3, b_5, c_7}$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>${a_3, b_3}$</td>
<td>${a_1, b_1}$</td>
<td>${b_5}$</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>${b_5}$</td>
<td>${b_1}$</td>
<td>${b_3}$</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>${c_7}$</td>
<td>${c_1}$</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>${d_{10}}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Node</th>
<th>In</th>
<th>Out</th>
<th>ACCKillIn</th>
<th>AccKillOut</th>
<th>ForkKill</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>${a_1, b_1, c_1}$</td>
<td></td>
<td>${a_3, b_3, b_5, c_7}$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>${a_1, b_1, c_1}$</td>
<td>${a_1, b_1, c_1}$</td>
<td>${a_3, b_3, b_5, c_7}$</td>
<td></td>
<td>${a_3, b_3, b_5, c_7}$</td>
</tr>
<tr>
<td>3</td>
<td>${a_1, b_1, c_1}$</td>
<td>${a_3, b_3, c_1}$</td>
<td></td>
<td>${a_1, b_1}$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>${a_1, b_1, c_1}$</td>
<td>${a_1, b_1, c_1}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>${a_1, b_1, c_1}$</td>
<td>${a_1, b_5, c_1}$</td>
<td></td>
<td>${b_1}$</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>${a_1, b_1, c_1}$</td>
<td>${a_1, b_1, c_1}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>${a_1, b_1, c_1}$</td>
<td>${a_1, b_1, c_7}$</td>
<td></td>
<td>${c_1}$</td>
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<tr>
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<td>${a_1, b_1, c_1, c_7}$</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>${a_1, b_5, c_1, c_7}$</td>
<td>${a_1, b_5, c_1, c_7}$</td>
<td></td>
<td>${b_1}$</td>
<td>${b_1}$</td>
</tr>
<tr>
<td>10</td>
<td>${a_3, b_5, c_1, c_7}$</td>
<td>${a_3, b_3, b_5, c_1, c_7, d_{10}}$</td>
<td>${a_1, b_1}$</td>
<td>${a_1, b_1}$</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 8:** Data Flow Sets for one iteration on the parallel program in Figure 7.
predecessors) of the node. The reaching definition information, i.e., the \( \text{In} \) set at each node, is defined by the fixpoint of the equations in Figure 7.

For the parallel program given in Figure 6, the \( \text{In} \), \( \text{Out} \), \( \text{Gen} \), \( \text{Kill} \), \( \text{ParallelKill} \) and the accumulated kill sets are given in Figure 8. The system of equations converges on the second iteration. The figure shows the first iteration (which is the same as the second). Note that \( \text{ACKKillout} (10) \) contains \( b_1 \). This indicates that \( b_1 \) is killed by one or more of the parallel branches – in this case, it is killed by both sections A and B (via Section B1). By comparison, even though 'c' is defined in node 7, the definition is conditional on 'P', and thus \( c_1 \) does not appear in \( \text{ACKKillout} (10) \). The set \( \text{Out} (10) \) contains definitions \( b_3 \) and \( b_5 \), indicating a potential anomaly. In the case of 'b', this is an actual anomaly.

In the rest of this section, we define the above data flow analysis framework (PRDEF) formally and prove that this framework forms a monotone data flow system, an important criteria for the system to reach a fix point.

5.1 Proof showing that PRDEF is an MDFS

The data flow analysis framework for computing the reaching definitions information in parallel programs with the Parallel Sections construct, PRDEF is defined as the triple \( \langle L, \land, \mathcal{F} \rangle \), where, \( L \) is lattice whose elements are in the power set of the set of definitions in the program, \( \land \) is the meet operator, in our case, set intersection and \( \mathcal{F} \) is a function space consisting of transfer functions in the data flow framework, i.e., \( \mathcal{F} \) consists of functions of the following forms:

\[
\begin{align*}
F1: & \quad f_v(x) = (x - \text{Kill}(v) - \text{ParallelKill}(v)) + \text{Gen}(v) \\
F2: & \quad l_v(x) = (x + \text{Kill}(v)) - \text{Gen}(v) \\
F3: & \quad h_v(x) = (x + \text{Kill}(fork(v))) - \text{Gen}(fork(v)) \\
F5: & \quad m_v(x) = l_v(x) + (h_v(y) - f_v(z))
\end{align*}
\]
where, \( fork(v) \) refers to the fork node corresponding to the parallel block in which this node appears. Clearly, we can relate each of the above equations to the equations in Figure 7. \( f_v \) corresponds to the \( Out \) set and the argument \( x \) to this function is the corresponding \( In \) set. \( l_v \) corresponds to the \( ACCKillout \) set when the node \( v \) is neither a fork node nor a join node. \( h_v \) corresponds to the \( ForkKill \) set and finally, \( m_v \) corresponds to the \( ACCKillout \) set when the node \( v \) is a join node. In the equation for \( m_v \), we have different arguments for functions \( h_v \) and \( f_v \) since they are both different transfer functions and need not take \( x \) as their argument when \( x \) is the argument to \( m_v \). The argument \( y \) corresponds to the \( ACCKillin \) set of \( fork(v) \) and \( z \) corresponds to \( In \) set of \( v \). Therefore, the function \( m_v \) can be written as the projection of the first of the function \( p_v \), where \( p_v \) is a function whose domain and range is a set of triples of definitions (the \( T \) set). The lattice associated with the function space \( P \) (consisting of functions of the form \( p_v \)) has elements from the power set of \( T \). \( p_v \) is defined as follows:

\[
p_v((x, y, z)) = ((l_v(x) + (h_v(y) - f_v(z))), h_v(y), f_v(z))
\]

Clearly, \( m_v(x) \) is the first element of \( p_v((x, y, z)) \). In the rest of the paper, we refer to \( L \) as the lattice whose elements are from the power set of the set of definitions in the program and \( L_1 \) as the lattice whose elements are from the power set of \( T \). The theorems that follow prove that each of the function spaces mentioned above are monotone.

**Theorem 2** The function space \( F1 \) associated with the lattice \( L \) is a monotone function space.

*Proof:* Each \( f \in F1 \) is of the form:

\[
f_v(x) = (x - a) + g
\]

To prove that \( F1 \) is a monotone function space, we will prove each of the properties listed in Definition 2.

**M1:** For any \( x, y \in L \) such that, \( x \subseteq y \), it is clear that \( (x - a) \subseteq (y - a) \). Therefore,

\[
((x - a) + g) \subseteq ((y - a) + g) \text{ or, } f_v(x) \subseteq f_v(y). \quad \square
\]

**M2:** There is an identity function, \( e(x) = (x - a) + g \) such that \( a = g = \emptyset \) that satisfies the property, \( e(x) = x \). \quad \square
M3: To prove that $F1$ is closed under composition, consider functions $f_1$ and $f_2$ in the function space $F1$. Let,

$$f_1(x) = (x - a_1) + g_1 \text{ and } f_2(x) = (x - a_2) + g_2.$$  

In bit vector notation,

$$f_1(x) = (x \land \overline{a_1}) \lor g_1 \text{ and } f_2(x) = (x \land \overline{a_2}) \lor g_2.$$  

Therefore, the composition of $f_1$ and $f_2$ is

$$f_1(f_2(x)) = (x \land \overline{a_3}) \lor g_3 \text{ where,}$$

$$\overline{a_3} = \overline{a_1} \land \overline{a_2} \text{ and } g_3 = (g_2 \land \overline{a_1}) \lor g_1.$$  

M4: For every $x \in L$, we can always find a function $f \in F1$ such that, $x = f_v(\bot)$. Such an $f$ has the following definition:

$$f_v(y) = (y \land \overline{a}) \lor g), \text{ where } \overline{a} = 0, \text{ and } g = x.$$  

$\square.$

Theorem 3  The function space $F2$ associated with the lattice $L$ is a monotone function space.

Proof: The proof of this theorem is similar to the proof of theorem 2. Functions in $F2$ are of the form:

$$l_v(x) = (x + k) - g$$

We proceed by proving each of the properties in Definition 2:

M1: For every $x, y \in L$, such that $x \subseteq y$, it is clear that $((x + k) - g) \subseteq ((y + k) - g)$ and hence, $l_v(x) \subseteq l_v(y). \square$

M2: The identity function, $e(x) = (x + k) - g$ that satisfies the property, $e(x) = x$ can be derived by choosing $k = g = \emptyset.$  \square

M3: Let $l_1, l_2 \in F2$, such that

$$l_1(x) = (x + k_1) - g_1 \text{ and } l_2(x) = (x + k_2) - g_2, \text{ i.e.,}$$

$$l_1(x) = (x \lor k_1) \land \overline{g_1} \text{ and } l_2(x) = (x \lor k_2) \land \overline{g_2}.$$
Therefore, \( l_1(l_2(x)) = (x \lor k_3) \land \bar{g}_3 = l_3(x) \), where,
\[
k_3 = k_1 \lor k_2 \quad \text{and} \quad \bar{g}_3 = (\bar{g}_2 \lor k_1) \land \bar{g}_1.
\]

Hence, \( F2 \) is closed under function composition. □

M4: For any given \( x \), we can always choose a function \( l \in F2 \), such that, \( l_0(y) = (y \lor k) \land \bar{g} \), where \( k = x \) and \( \bar{g} = x \). Therefore, \( x = l_0(y) \) and, in particular, \( x = l_0(\bot) \). □

**Theorem 4**  The function space \( F3 \) associated with the lattice \( L \) is a monotone function space.

*Proof:* Functions in \( F3 \) are of the form:
\[
h_v(x) = (x + \text{Kill}(\text{fork}(v))) - \text{Gen}(\text{fork}(v))
\]

i.e., \( h_v(x) = (x + kf) - g f \). Since \( h_v \) is similar to \( l_v \), except for the constants in the two functions, the proof of this theorem is identical to the proof of theorem 3. □

**Theorem 5**  The function space \( F5 \) associated with the lattice \( L_1 \) is a monotone function space.

*Proof:* Recall the definition of a function in the function space, \( F5 \):
\[
p_v((x, y, z)) = (((l_v(x) + (h_v(y) - f_v(z))))_v, h_w(y), f_w(z))
\]

where, \( l_v(x) = (x + c1) - c2 \),
\( h_v(y) = (y + c3) - c4 \),
\( f_v(z) = (z - c5) + c6 \),
\( h_w(y) = ((y + c7) - c8) \),
and \( f_w(z) = ((z - c9) + c10) \).

We now prove the four properties for monotonicity of the function space, \( F5 \):

M1: Consider some \( a, b \in L_1 \) such that \( a \leq b \).

i.e., \( a = (x_1, y_1, z_1) \) and \( b = (x_2, y_2, z_2) \) and \( x_1 \subseteq x_2, y_1 \subseteq y_2 \) and \( z_1 \subseteq z_2 \).

We know from theorems 2, 3 and 4 that \( l_v(x_1) \subseteq l_v(x_2), h_v(y_1) \subseteq h_v(y_2) \) and \( f_v(z_1) \subseteq f_v(z_2) \).
Therefore, \((l_u(x_1) + (h_w(y_1) - f_v(z_1))) \subseteq (l_u(x_2) + (h_w(y_2) - f_v(z_2)))\)

The proof that \(h_w(y_1) \subseteq h_w(y_2)\) and \(f_u(z_1) \subseteq f_u(z_2)\) are similar to the proofs of M1 in theorems 3 and 2 respectively.

Hence, \(p_v(a) \leq p_v(b)\). □

M2: The identity function \(e\) associated with \(L_1\) that satisfies the property, \(e((x, y, z)) = (x, y, z)\) is the following:

\[
e_v((x, y, z)) = ((l_u(x) + (h_u(y) - f_v(z)), h_w(y), f_u(z))
\]

where, \(l_u(x) = (x + c1) - c2\), such that, \(c1 = c2 = 0\),
\(h_u(y) = (y + c3) - c4\), such that, \(c3 = 0\) and \(c4 = y\).
\(f_u(z) = (z - c5) + c6\), such that, \(c5 = z\) and \(c6 = 0\),
\(h_w(y) = (y + c7) - c8\), such that, \(c7 = c8 = 0\), and,
\(f_u(z) = (z - c9) + c10\), such that, \(c9 = c10 = 0\).

Therefore, \(e_v((x, y, z)) = (x, y, z)\).

M3: Let \(p_1, p_2 \in F_5\) such that,

\[
p_1((x, y, z)) = ((l_1(x) \lor (h_1(y) \land \overline{f_1(z)}), h_{w_1}, f_{z_1})
\]

where \(h_{w_1} = ((y \lor c7') \land \overline{c8'}))\) and \(f_{u_1} = ((z \land \overline{c9'}) \lor c10')\) and,
\[
p_2((x, y, z)) = ((l_2(x) \lor (h_2(y) \land \overline{f_2(z)}), h_{w_2}, f_{z_2})
\]

where \(h_{w_2} = ((y \lor c7'') \land \overline{c8''}))\) and \(f_{u_2} = ((z \land \overline{c9''}) \lor c10'')\).

Therefore, the composition, \(p_1(p_2((x, y, z)))\) is defined as

\[
((l_3(x) \lor (h_3(y) \land \overline{f_3(z)}), h_{w_3}, f_{u_3}(z))
\]

where, \(l_3(x) = (x \lor c1) \land \overline{c2}; c1 = c1' \lor c1''\) and \(c2 = (c2' \lor c2'')\),
\(h_3(y) \land \overline{f_3(z)} = (h'(y) \land \overline{f'(z)}) \lor (h''(y) \land \overline{f''(z)})\); where
\(h'(y) = h_2(y) \land h_1(h_{w_2}(y)); f'(z) = f_2(z) \lor f_1(f_{u_2}(y)),
\)
\(h''(y) = h_2(y) \lor h_1(h_{w_3}(y)); f''(z) = c2'' \lor (f_2(z) \lor f_1(f_{u_2}(z))),
\)
\(h_{w_3}(y) = ((y \lor c7) \land \overline{c8}); c7 = c7' \lor c7''\) and \(c8 = (c8' \lor c8'')\).

and \(f_{u_3}(z) = ((z \land \overline{c9}) \lor c10); c9 = c9' \land c9''\) and \(c10 = (c10'' \land \overline{c9'}) \lor c10'\). □
M4: For any given \( a = (x_1, y_1, z_1) \in L_1 \), we can always find a function \( p_v \in F5 \) such that, 
\( a = p_v(\perp) \).

Simply make the following substitutions in the functions \( p_v \),
\[
p_v((x, y, z)) = ((l_v(x) + (h_v(y) - f_v(z))), h_w(y), f_u(z))
\]
where, \( l_v(x) = (x + c1) - c2; \ c1 = c2 = x_1 \), 
\( h_v(y) = (y + c3) - c4; \ c3 = \emptyset, c4 = y \), 
\( f_v(z) = (z - c5) + c6; \ c5 = z, c6 = \emptyset, \)
\( h_w(y) = ((y + c7) - c8); \ c7 = y_1, c8 = y_1 \), and 
\( f_u(z) = ((z - c9) + c10); \ c9 = \emptyset, c10 = z_1. \)
\( \square \)

**Corollary 1** The function space \( F4 \) is a monotone function space associated with the lattice \( L \). Note that each function in \( F4 \) can be defined as the projection of the first element of a corresponding function in \( F5 \).

**Theorem 6** PRDEF is a monotone data flow system.

*Proof:* Clearly the lattice, \( L \) is a semilattice defined on elements in the power set of the set of definitions in the program with meet operator as set intersection and \( \perp = \emptyset \). From the theorems 2, 3, 4 and corollary 1, we know that the function spaces \( F1, F2, F3 \) and \( F4 \) are monotone. Therefore, the function space \( F \) in PRDEF is monotone. Hence, the theorem follows by definition of an MDFS. \( \square \)

6 Including the effect of Synchronization

We extend the data flow equations in the previous section to consider event synchronization by using the preserved sets formulation given in [3]. Synchronization using post/wait occurs between different threads that execute in parallel. Synchronization edges carry data flow information, i.e., they propagate values of variables from the thread that posted the event to the thread that is waiting for the event to be posted. According to [8], it must be possible to execute
each post before its corresponding wait for a parallel program to be deadlock free and correct. If the post statement at a node \( n_p \) always executes before the corresponding wait node, \( n_w \)\(^2\), then \( n_w \) will have to update its reaching definitions information with that from the Out set of the node corresponding to the post. Apart from updating the reaching definitions information, i.e., the In set at the wait node, \( n_w \), we also want to be able update its accumulated kill sets, e.g., if \( n_w \) defines a variable, then all definitions of that variable reaching \( n_w \) via synchronization edges from post nodes, \( n_p \), that always execute before \( n_w \), must be included in the ACCKillin set of \( n_w \). This is important because the definitions propagated by such synchronization edges are killed by the corresponding definition in \( n_w \).

If there is a synchronization edge from \( n_p \) to \( n_w \), we can not say that \( n_p \) always executes before \( n_w \). It is possible that there are multiple posts of the same event variable and multiple waits for the same event variable. It is also possible that these multiple posts and waits are executed conditionally. Thus, a synchronization edge does not always imply an execution order. We are, however, interested in the potential execution order for computing the reaching definition information. Preserved sets, as defined in [3] give precisely the set of nodes that execute before a given node, defined as follows:

**Definition 3** A node \( n_j \in \text{Preserved}(n_i) \) if and only if for all parallel executions \( x \), if \( n_j \) and \( n_i \) are both executed, \( n_j \) is completed before \( n_i \) is begun.

However, Callahan and Subhlok [3] have shown that computing this information is Co-NP Hard and have given a data flow framework to compute a conservative approximation to the Preserved sets. The approximate Preserved sets are computed as the least fixpoint of a set of data flow equations over the Parallel Flow Graph. The Preserved set for a block is defined as a function of its control flow (parallel and sequential) and synchronization predecessors.

Clearly, by using the Preserved set formulation, we can determine at a wait node \( n_w \) if a post node, \( n_p \), always completes execution before \( n_w \) begins, i.e., if \( n_p \in \text{Preserved}(n_w) \). We use a new data flow set, called the SynchPass set, that propagates definitions via synchronization edges.

If \( n_p \) executes before \( n_w \), we propagate the definitions from \( n_p \) to \( n_w \) (and thus all nodes that execute after \( n_w \) in the same thread), because we know those definitions must have oc-

\(^2\)We say a wait node starts executing when the wait statement is successful and the code following the wait in this node starts execution.
Figure 9: Synchronization Example

curred before the synchronization occurred. Any definitions that occur in node $n_w$ (and nodes subsequently executed by that thread) will kill the previous definitions in the thread executing $n_w$. These definitions will also kill any definitions that occur before $n_p$ executes in the thread corresponding to $n_p$, but not necessarily those definitions occurring after $n_p$ executed in that thread (e.g., because the thread executing $n_p$ may have already completed execution, as it does not wait for the wait statement to occur). This means that the join node must realize that the definitions in $n_w$ occur after the definitions passed in from $n_p$; this is the role of the $ACCKill$ sets.

For example, consider the Parallel Sections in the PFG shown in Figure 9. The fork node defines a value for ‘x’. This value reaches the predecessor of the wait node and the post node. The definition in the fork node is in the $ACCKillout$ set for the post node, indicating that some branch of the Parallel Section has killed that value. However, only the value from the wait node should reach the join node, because that definition must occur after the assignment in the post node and the fork node. We get the execution ordering information from the Preserved set. The value of ‘Y’ following the post node is not specified by the language
$\text{SynchPass}(n) = \begin{cases} \bigcup \limits_{p \in \text{synch} \_ \text{pred} \land p \in \text{Preserved}(n)} \text{Out}(p) \\ \bigcup \limits_{p \in \text{par} \_ \text{pred}} \text{SynchPass}(p) + \bigcap \limits_{p \in \text{seq} \_ \text{pred}} \text{SynchPass}(p) \end{cases}$ (if $n$ is a wait node)

\begin{align*}
\text{Out}(n) &= \bigg\{ (\text{In}(n) - \text{Kill}(n) - \text{ParallelKill}(n) \cup \text{Gen}(n)) - \\
&\quad (\text{OtherDefs}(n) \cap \text{SynchPass}(n)) \bigg\}
\end{align*}

\begin{align*}
\text{In}(n) &= \bigg\{ \bigcup \limits_{p \in \text{pred}(n)} \text{Out}(p) - \bigcup \limits_{p \in \text{par} \_ \text{pred}(n)} \text{ACCKillout}(p) - \\
&\quad \bigcap \limits_{p \in \text{synch} \_ \text{pred}(n)} \text{ACCKillout}(p) \bigg\}
\end{align*}

\begin{align*}
\text{ACCKillout}(n) &= \begin{cases} \emptyset & (n \text{ is a fork node}) \\
(\text{ACCKillin}(n) + \text{Kill}(n)) - \text{Gen}(n) & (n \text{ is a join node, with corresponding fork node } f) \\
+ (\text{ForkKill}(f) - \text{Out}(n)) \\
(\text{ACCKillin}(n) + \text{Kill}(n)) - \text{Gen}(n) & (\text{otherwise})
\end{cases}
\end{align*}

\begin{align*}
\text{ACCKillin}(n) &= \bigg\{ \bigcup \limits_{p \in \text{par} \_ \text{pred}(n)} \text{ACCKillout}(p) + \\
&\quad \bigcap \limits_{p \in \text{seq} \_ \text{pred}(n)} \text{ACCKillout}(p) \\
&\quad + (\text{OtherDefs}(n) \cap \text{SynchPass}(n)) \bigg\}
\end{align*}

\begin{align*}
\text{ForkKill}(n) &= \begin{cases} (\text{ACCKillin}(n) + \text{Kill}(n)) - \text{Gen}(n) & (n \text{ is a fork node}) \\
\emptyset & (\text{otherwise})
\end{cases}
\end{align*}

**Figure 10:** Dataflow Equations for Programs with Parallel Sections and Event Synchronization
<table>
<thead>
<tr>
<th>Node</th>
<th>Gen</th>
<th>Kill</th>
<th>ParKill</th>
</tr>
</thead>
<tbody>
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<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
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<td>{x_8}</td>
</tr>
<tr>
<td>5</td>
<td>{x_5}</td>
<td>{x_0, x_4}</td>
<td>{x_8}</td>
</tr>
<tr>
<td>6</td>
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<td></td>
<td>{z_9}</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
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<td>{x_0}</td>
<td>{x_4, x_5}</td>
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<tr>
<td>9</td>
<td>{z_9}</td>
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<td>{x_0}</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>{y_{11}}</td>
<td>{y_0}</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Node</th>
<th>In</th>
<th>Out</th>
<th>ACCKillIn</th>
<th>AccKillOut</th>
<th>ForkKill</th>
</tr>
</thead>
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<td></td>
</tr>
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<td>{x_0, y_0}</td>
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<td></td>
</tr>
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<td>{x_0}</td>
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</tr>
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<tr>
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<td>{x_0, x_4, x_5}</td>
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**Figure 11**: Data Flow Sets for the program in Figure 4 : Iteration 1.
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<tr>
<th>Node</th>
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<th>Out</th>
<th>ACCKillIn</th>
<th>AccKillOut</th>
<th>ForkKill</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>{x_0, x_8, y_0, y_{11}, z_6, z_9}</td>
<td>{x_0, x_8, y_0, y_{11}, z_6, z_9}</td>
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<td>{x_4, x_5}</td>
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<td>{x_0, x_8, y_0, y_{11}, z_6, z_9}</td>
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<tr>
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<td>8</td>
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<td></td>
</tr>
</tbody>
</table>

**Figure 12:** Data Flow Sets for the program in Figure 4: Iteration 2.
definition. One could argue that 'Y' should have the value '3'; however, we have chosen to assume copy-in/copy-out semantics, and would thus believe that 'Y' has the value '2'—ideally, an error message would be issued concerning this data race. For this example, the data flow formulation for Preserved sets given in [3] will be able to determine the Preserved sets of the wait node accurately. However, since this data flow formulation is conservative, we may not be always able to compute the exact Preserved sets for any node. This would result in a conservative approximation to the reaching definitions information in our data flow framework. For example, in the absence of the Preserved sets information in figure 9, we would derive the Out set of the join node to contain the definitions from both the post and the wait node. This is a conservative, yet correct, approximation to the reaching definition information at the join node. In the worst case, the effect of synchronization is lost at parallel merge points, i.e., in the absence of any Preserved set information our data flow equations would compute multiple reaching definitions at respective parallel merge nodes. This simply reduces the opportunity or effectiveness of some optimizations.

Therefore, at wait nodes, we update the SynchPass set with the Out set from the corresponding synchronization predecessors in the Preserved set of this node, indicating that the definitions from those predecessors have occurred. In order to propagate the SynchPass information to other nodes after a wait node, we want to consider the union of the SynchPass from all the parallel predecessors (since all these predecessors always execute) and the intersection of the SynchPass from the sequential predecessors (since only one of them executes).

We update the ACCKillin set of each node with the definitions of variables that are propagated by synchronization edges (i.e. SynchPass). We only consider the definitions of SynchPass also defined in this node. To do this, we use the set OtherDefs (n), or the definitions in the program outside of n that define variables that also have definitions within n.

The data flow equations taking into account Parallel Sections constructs with event synchronization is given in Figure 10. In this figure, synch_pred refers to synchronization predecessor.

Figures 11 and 12 show the data flow sets for the first two iterations for the parallel program in Figure 3; the fix point is reached in the third iteration. The Preserved set of node (8) (the wait node) is the set {Entry, 1, 2, 3, 4, 5, 7}, since each of these nodes always completes execution before node (8), if they execute at all. The reaching definition information in this
figure has been computed using the Preserved set information. The definitions, \( x_4 \) and \( x_5 \)
will not reach the join node, (11), because the definition \( x_8 \) always executes after \( x_4 \) and \( x_5 \).
It is this information on execution order that we borrowed from the Preserved set formulation.
Also, the \( ACCK\)illout set of (11) includes \( x_4 \) and \( x_5 \). This information was propagated to
node (8) by the synchronization edges since (4) and (5) were in the Preserved set of (8). The
definitions \( z_6 \) and \( z_9 \) reach the merge node (11); this is an indication of a potential anomaly
in the program since the two definitions occur in distinct parallel branches, i.e., threads that can
execute in parallel. The importance of the \( ParallelKill \) set is seen in the \( Out \) set of nodes (6)
and (9). Even though the corresponding \( In \) sets have both definitions of \( z' \), only the definition
in that node should be in its \( Out \) set. The reason the \( In \) set of (6) and (9) both have \( z_6 \)
and \( z_9 \) is because of the loop around the parallel block. Since we exclude the \( ParallelKill \) set
from the \( Out \) set, we are able to compute the correct \( Out \) sets; for example, the \( Out \) set of
(6) does not contain \( z_9 \) since this definition is in its \( ParallelKill \) set.

**Theorem 7** The Data Flow System described in Figure 10 is an MDFS.

**Proof:** The computation of \( SynchPass \) set is similar to that of the \( In \) set, i.e., \( SynchPass \) is
a synthesized attribute in the data flow system. The only transfer function that gets modified
as a result of the \( SynchPass \) set is the transfer function for the \( Out \) set. We can represent this
transfer function in terms of a function, \( q_v \), whose domain and range is a set of definition pairs:

\[
q_v((x, y)) = (((x - a + g) - (d \cap y), (e \cap y) + f))
\]

where \( a = Kill(v) + ParallelKill(v) \) and
\( g = Gen(v) \), \( d = OtherDefs(v) \) and

constants \( e \) and \( f \) can be chosen to be any constant set for purposes of the proof.

The required function is then the projection of the first element of the result of \( q_v((x, y)) \).
The detailed proof is similar to that in theorem 5 and is not repeated here. The proof would
proceed by showing that the function space \( F_q \), where \( q_v \in F_q \), is monotone, followed by
implying that the projection of the first of all the \( q_v \)'s in \( F_q \) also forms a monotone function
space.

\( \square \)
7 Conclusion

We have presented data flow equations from [5] to compute the reaching definition information at any point in an explicitly parallel program. Data flow equations for computing reaching definitions information in sequential programs have been well understood and used in many current day compilers for the optimization of such programs. We believe that the data flow framework that we have presented in this paper can be used to perform rigorous scalar optimization on parallel programs and thus help achieve better execution rates of such programs on existing high performance architectures. This information will particularly benefit distributed shared memory systems, because optimizations using the data flow information will reduce the amount of communication between processors.

We have also presented proofs showing that the data flow framework to compute the reaching definitions information in explicitly parallel programs is a monotone data flow system. We have considered parallel programs with post/wait synchronization. The Preserved set formulation would be different for other synchronization constructs. However, we do not anticipate the data flow framework presented in this paper to change for other synchronization constructs.

References


