P1. Consider the following traces ($\omega$-strings) involving systems with state variables $x, y$

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>0</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>0</th>
<th>-1</th>
<th>1</th>
<th>1</th>
<th>(-1</th>
<th>1)(\omega)</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>-1</td>
<td>2</td>
<td>2</td>
<td>(-1</td>
<td>2)(\omega)</td>
<td>...</td>
</tr>
</tbody>
</table>

For each of the temporal formulas below, explain whether or not they hold on the trace. Write a short justification.

(a) $\Box(x > 0)$
- **False.** We reach state $x = 0$ at second step.

(b) $\Diamond(x \leq 0)$
- **True.** We reach state $x = 0$ at second step.

(c) $\Diamond\Box(y \leq x)$
- **False.** The state $y = 2, x = 1$ is reached infinitely often in the trace and does not satisfy the atom $y \leq x$.

(d) $\Box\Diamond(y = x)$.
- **True.** The state $x = y = -1$ is reached infinitely often.

(e) $\Diamond\Box(|x - y| \leq 1)$
- **True.** In fact $\Box(|x - y| \leq 1)$ is true starting from the very start of the trace.

(f) $(y \leq x)$
- **True.** This is an atomic formula. It is true at the first step and therefore considered true for the whole trace.

(g) $(y \leq x) \mathcal{U} (y = 2)$
- **True.** The proposition $y \leq x$ holds initially until we get to the 7th step where $y = 2$.

(h) $(|x - y| \leq 1) \mathcal{U} (|x - y| = 2)$.
- **False.** $|x - y| = 2$ is never reached in the trace.

(i) $\Diamond(x - y = 1)$.
True. $x - y = 1$ holds at the second step of the trace.

P2 Draw $\omega$-automata for formulae (a), (c), (e), (g) and (i) above.

(a) $\Box (x > 0)$

(c) $\Diamond \Box (y \leq x)$

(e) $\Diamond \Box (|x - y| \leq 1)$

(g) $(y \leq x) \cup (y = 2)$
(i) $\bigcirc (x - y = 1)$.

P3 Consider the following system:

state $x$ : int initially 0;
state $y$ : int initially 0;
if ($x == 1$)
  $x := 0$; $y := 0$
else
  $x := 1$; $y := -1$;

For each of the formulae (a) - (i) in P1, explain whether they hold on the system or not with a short justification.

The program just generates a single trace shown below as a sequence of $(x, y)$ values:

$$(0, 0) (1, -1) (0, 0) (1, -1) \cdots$$

(a) $\Box(x > 0)$

**False.** We reach state $x = 0$ at first step.

(b) $\Diamond(x \leq 0)$

**True.** We reach state $x = 0$ at first step.
(c) $\Diamond \Box (y \leq x)$

- **True.** In fact $\Box (y \leq x)$ is true from the very first state.

(d) $\Box \Diamond (y = x)$.

- **True.** The state $x = y = 0$ is reached infinitely often.

(e) $\Diamond \Box (|x - y| \leq 1)$

- **False.** Every $(1,-1)$ state violates this and happens infinitely often.

(f) $(y \leq x)$

- **True.** This is an atomic formula. It is true at the first step and therefore considered true for the whole trace.

(g) $(y \leq x) \mathcal{U} (y = 2)$

- **False.** $y = 2$ is never reached.

(h) $(|x - y| \leq 1) \mathcal{U} (|x - y| = 2)$.

- **True.** The very first state satisfies $|x-y| \leq 1$ and the very second state satisfies $|x-y| = 2$.

(i) $\Diamond (x - y = 1)$.

- **False.** $x - y = 1$ does not at the second step of the trace.