Verification and Control of CPS: Assignment 4
Due Date: Thursday, October 23rd, 2014 (4:30 PM)

P1. Consider the following traces (ω-strings) involving systems with state variables x, y

x	1	0	1	0	-1	0	1	0	-1	1	-1	1	(-1	1)$^\omega$	$\cdots$
y	0	-1	0	-1	-1	0	2	0	-1	2	-1	2	(-1	2)$^\omega$	$\cdots$

For each of the temporal formulas below, explain whether or not they hold on the trace. Write a short justification.

(a) $\Box(x > 0)$

False. We reach state $x = 0$ at second step.

(b) $\Diamond(x \leq 0)$

True. We reach state $x = 0$ at second step.

(c) $\Diamond\Box(y \leq x)$

False. The sate $y = 2, x = 1$ is reached infinitely often in the trace and does not satisfy the atom $y \leq x$.

(d) $\Box\Diamond(y = x)$.

True. The state $x = y = -1$ is reached infinitely often.

(e) $\Diamond\Box(|x - y| \leq 1)$

True. In fact $\Box(|x - y| \leq 1)$ is true starting from the very start of the trace.

(f) $(y \leq x)$

True. This is an atomic formula. It is true at the first step and therefore considered true for the whole trace.

(g) $(y \leq x)\ \mathcal{U}\ (y = 2)$

True. The proposition $y \leq x$ holds initially until we get to the 7th step where $y = 2$.

(h) $(|x - y| \leq 1)\ \mathcal{U}\ (|x - y| = 2)$.

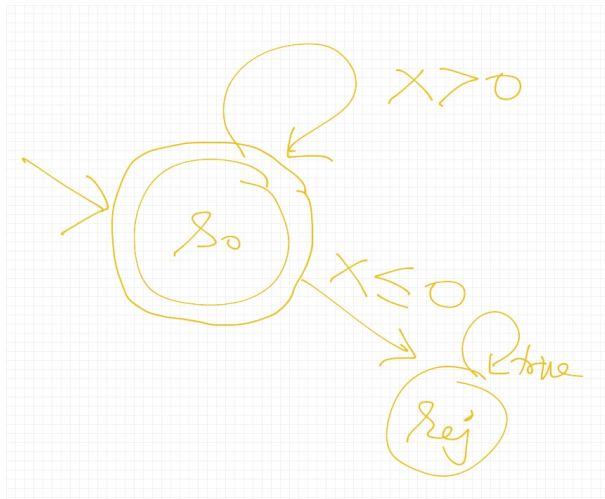
False. $|x - y| = 2$ is never reached in the trace.

(i) $\bigcirc(x - y = 1)$.

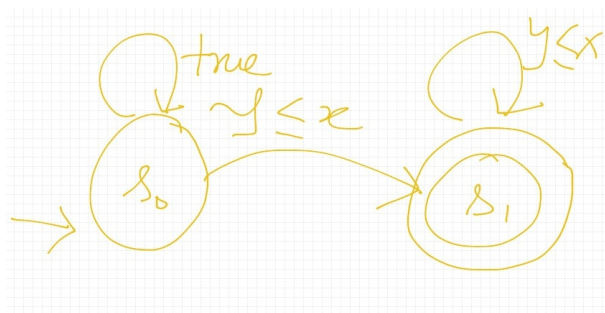
True. $x - y = 1$ holds at the second step of the trace.

P2 Draw ω-automata for formulae (a), (c), (e), (g) and (i) above.

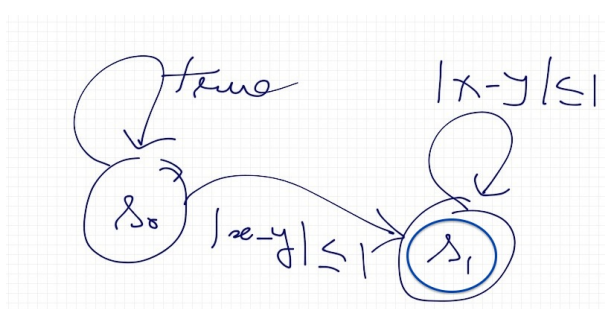
(a) $\Box(x > 0)$



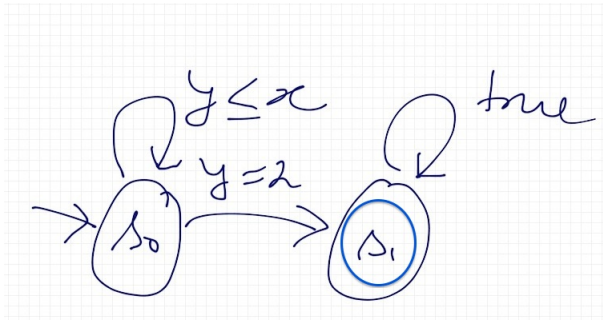
(c) $\Diamond\Box(y \leq x)$



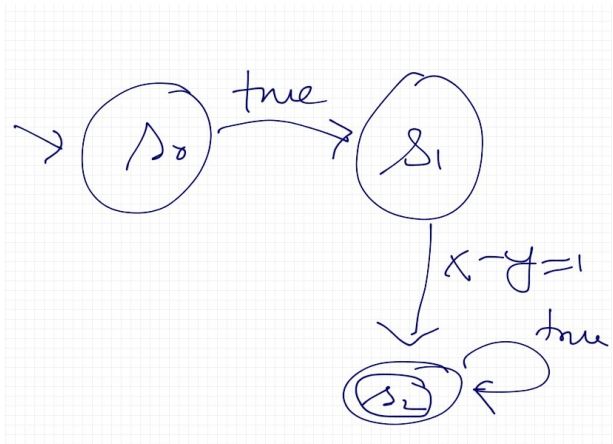
(e) $\Diamond\Box(|x - y| \leq 1)$



(g) $(y \leq x)\ \mathcal{U}\ (y = 2)$



(i) $\bigcirc(x - y = 1)$.



P3 Consider the following system:

```
state x : int initially 0;
state y : int initially 0;
    if ( x == 1)
        x := 0; y := 0 ;
    else
        x := 1; y := -1;
```

For each of the formulae (a) - (i) in P1, explain whether they hold on the system or not with a short justification.

The program just generates a single trace shown below as a sequence of (x, y) values:

$$(0,0)\ (1,-1)\ (0,0)\ (1,-1)\ \cdots$$

(a) $\Box(x > 0)$

False. We reach state $x = 0$ at first step.

(b) $\Diamond(x \leq 0)$

True. We reach state $x = 0$ at first step.

(c) $\Diamond\Box(y \leq x)$

True. In fact $\Box(y \leq x)$ is true from the very first state.

(d) $\Box\Diamond(y = x)$.

True. The state $x = y = 0$ is reached infinitely often.

(e) $\Diamond\Box(|x - y| \leq 1)$

False. Every $(1, -1)$ state violates this and happens infinitely often.

(f) $(y \leq x)$

True. This is an atomic formula. It is true at the first step and therefore considered true for the whole trace.

(g) $(y \leq x)\ \mathcal{U}\ (y = 2)$

False. $y = 2$ is never reached.

(h) $(|x - y| \leq 1)\ \mathcal{U}\ (|x - y| = 2)$.

True. The very first state satisfies $|x-y| \leq 1$ and the very second state satisfies $|x-y| = 2$.

(i) $\bigcirc(x - y = 1)$.

False. $x - y = 1$ does not at the second step of the trace.