P1. Consider the synchronous components $C_1, C_2$ with their imperative description. For component $C_2$, note that input $i$ is an integer but can only take the values \{0, 1\}.

\[\begin{align*}
\text{state } x_1 &\text{: int initial 0} \\
\text{if } (i_1 == i_2): &\text{ else} \\
& o_1 = x_1; x_1 = 1 - x_1; \\
& o_1 = 1 - x_1; x_1 = 0;
\end{align*}\]

\[\begin{align*}
\text{state } x_2 &\text{: int initial 0} \\
& x_2 = (x_2 + i) \mod 3; \\
& \text{if } (x_2 == 2): o_2 = \text{true} \\
& \text{else } o_2 = \text{false}
\end{align*}\]

(A) Carefully draw state diagrams for the two components above. Draw just the reachable states of each component. You may check your answer for this part carefully with your classmates or the instructor before proceeding.

**Solution** For Component $C_1$
Draw state diagrams for the compositions shown below.

(B) Output of $C_1$ is fed as input of $C_2$.

Solution: See figure below. All green edges are labelled with input $(0, 0)$ or $(1, 1)$. All red edges are labelled with inputs $(0, 1)$ or $(1, 0)$. Furthermore, all outputs are false with the exception of the three edges labelled “t” whose outputs are true.
Suppose the input $i$ to the component $C_2$ were instead set to the range $\{-2, -1, 0\}$ argue whether the composition shown above is possible. Write a brief justification 2-3 sentences at most.

**Solution:** The interconnection is not possible. The output of $C_1$ belongs to $\{0, 1\}$ but the input of the new component $C_2$ does not accept 1. Therefore, the two components are incompatible and cannot be interconnected as shown.

(C) Output of $C_2$ is fed to both inputs of $C_1$.

(D) Parallel composition.
Solution: We will use the following legend for inputs on each edge. The outputs are shown next to each edge.

<table>
<thead>
<tr>
<th>Edge Type</th>
<th>Input ((i_1, i_2, i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green</td>
<td>((i_1 : 0, i_2 : 0, i : 0))</td>
</tr>
<tr>
<td></td>
<td>((i_1 : 1, i_2 : 1, i : 0))</td>
</tr>
<tr>
<td>Dashed Blue</td>
<td>((i_1 : 0, i_2 : 0, i : 1))</td>
</tr>
<tr>
<td></td>
<td>((i_1 : 1, i_2 : 1, i : 1))</td>
</tr>
<tr>
<td>Red</td>
<td>((i_1 : 1, i_2 : 0, i : 0))</td>
</tr>
<tr>
<td></td>
<td>((i_1 : 0, i_2 : 1, i : 0))</td>
</tr>
<tr>
<td>Orange</td>
<td>((i_1 : 1, i_2 : 0, i : 1))</td>
</tr>
<tr>
<td></td>
<td>((i_1 : 0, i_2 : 1, i : 1))</td>
</tr>
</tbody>
</table>

P2 After considering the compositions in each part of problem \(P1\ (B)-(D)\), write down at least two safety properties for each of the parts in \(P1\ (B)-(D)\) of the composed system that are true, and at least two safety properties that are violated by the composed system.

Solution: Examples of safety properties satisfied by all models in parts (B)-(D) are
(a) \(\square(x_1 \geq 0 \land x_2 \geq 0)\)
(b) \(\square(x_2 \leq 3)\)
Examples that are not satisfied by any of the three include
(a) \(\Box(x_1 = x_2)\)
(b) \(\Box(x_2 > x_1)\)