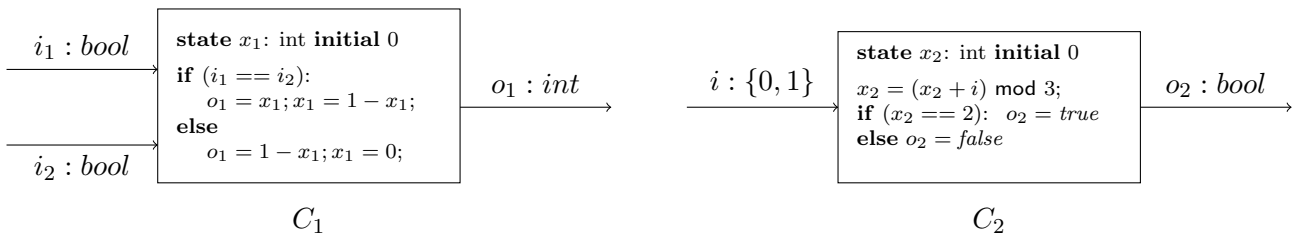
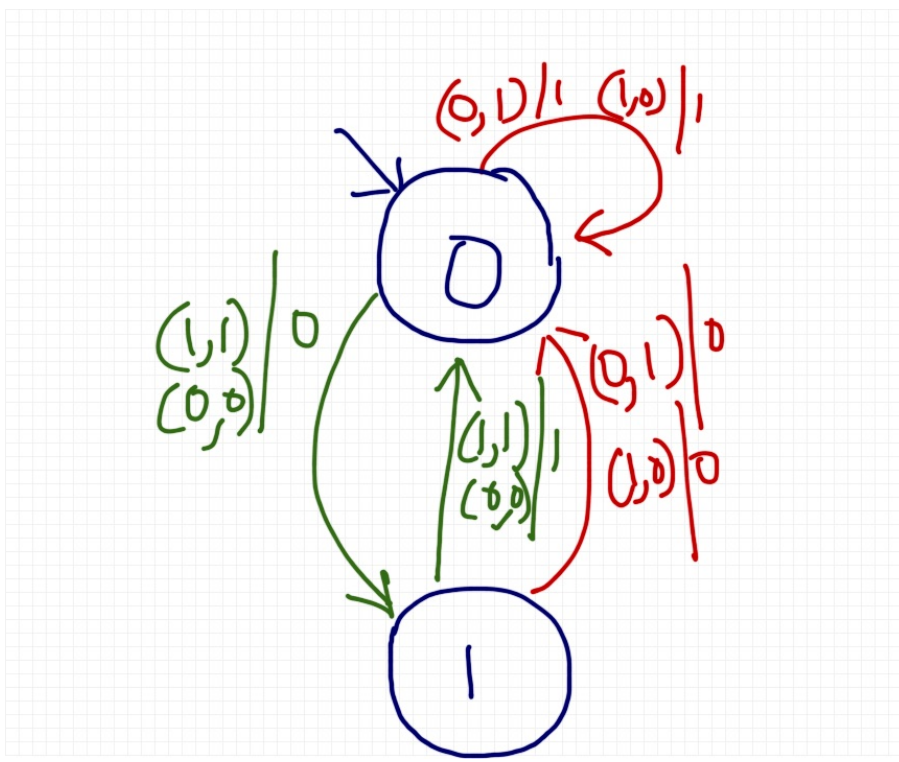
Verification and Control of CPS: Assignment 2
Due Date: Thursday, September 18, 2014 (4:30 PM)

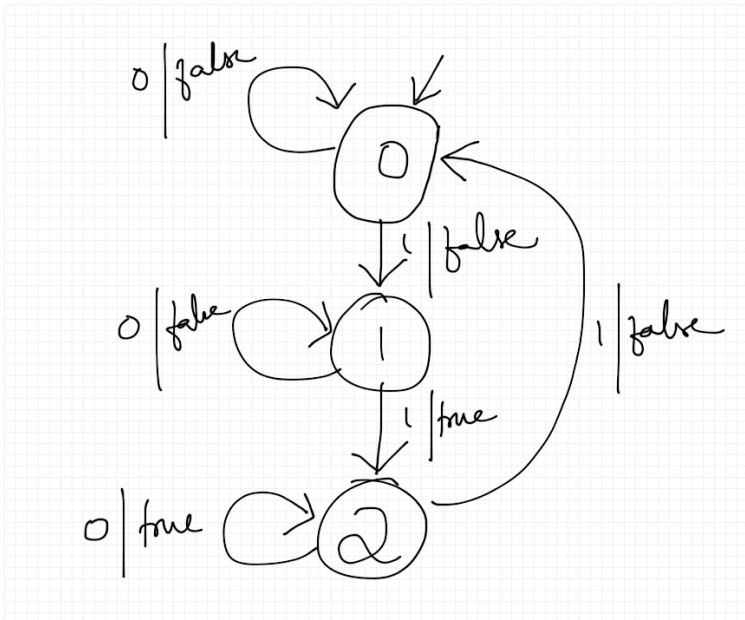
P1. Consider the synchronous components C_1, C_2 with their imperative description. For component C_2, note that input i is an integer but can only take the values $\{0, 1\}$.



(A) Carefully draw state diagrams for the two components above. Draw just the reachable states of each component. You may check your answer for this part carefully with your classmates or the instructor before proceeding.
Solution For Component C_1

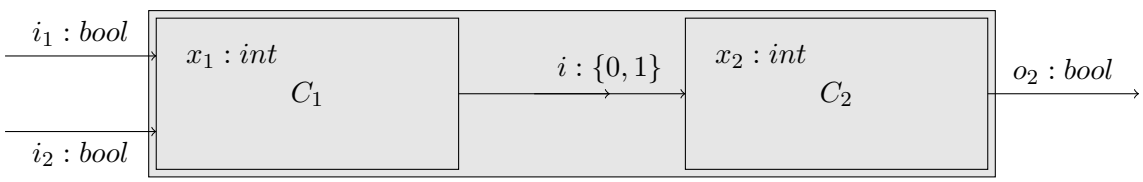


For Component C_2

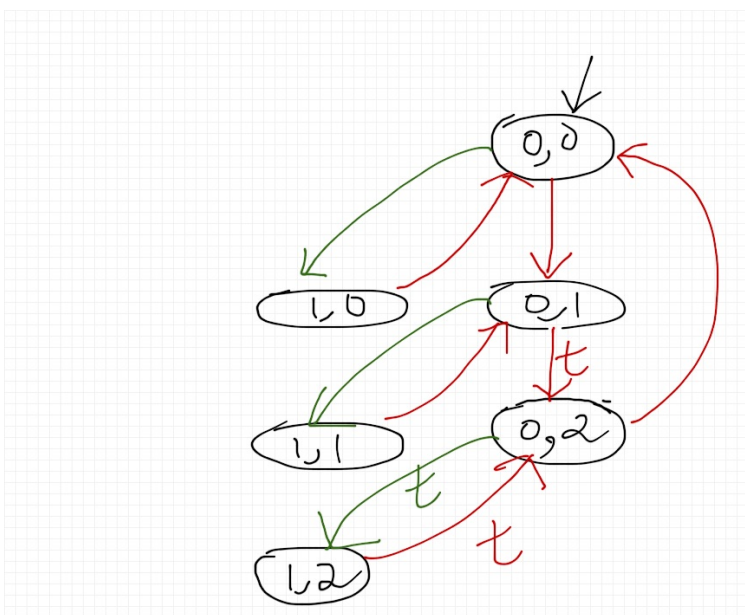


Draw state diagrams for the compositions shown below.

(B) Output of C_1 is fed as input of C_2.

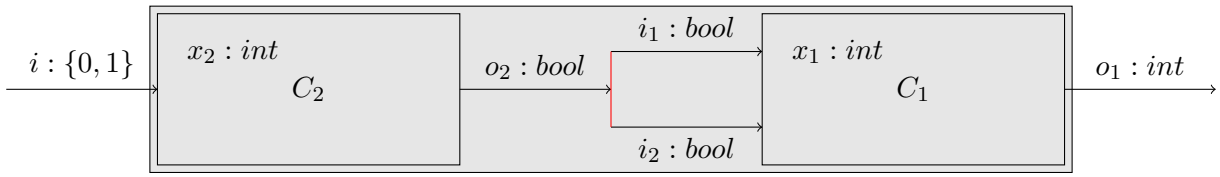


Solution: See figure below. All green edges are labelled with input $(0, 0)$ or $(1, 1)$. All red edges are labelled with inputs $(0, 1)$ or $(1, 0)$. Furthermore, all outputs are *false* with the exception of the three edges labelled "t" whose outputs are *true*.

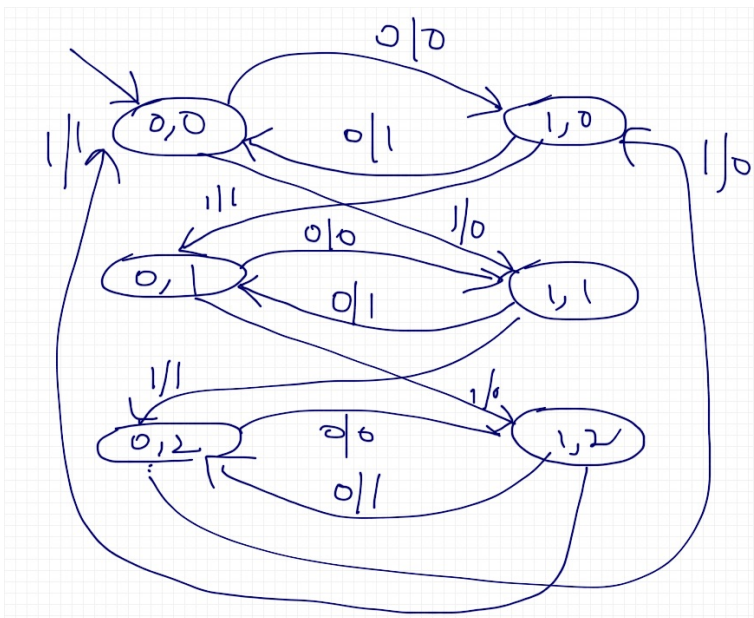


Suppose the input i to the component C_2 were instead set to the range $\{-2, -1, 0\}$ argue whether the composition shown above is possible. Write a brief justification 2-3 sentences at most.
Solution: The interconnection is not possible. The output of C_1 belongs to $\{0, 1\}$ but the input of the new component C_2 does not accept 1. Therefore, the two components are incompatible and cannot be interconnected as shown.

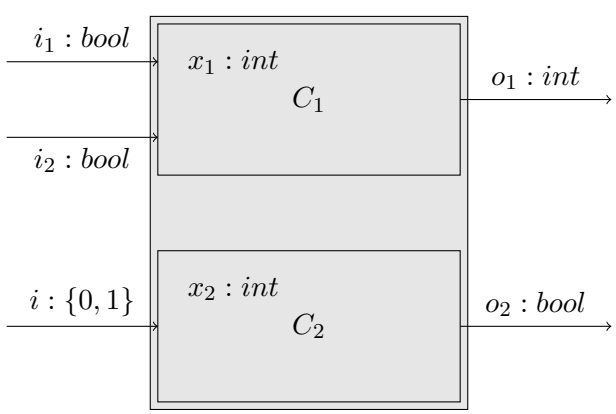
(C) Output of C_2 is fed to both inputs of C_1.



Solution:

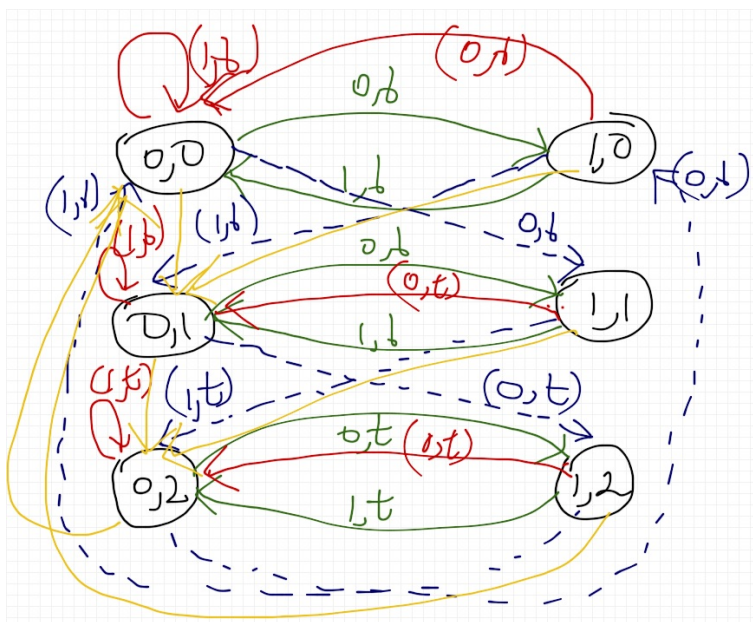


(D) Parallel composition.



Solution: We will use the following legend for inputs on each edge. The outputs are shown next to each edge.

Edge Type	Input (i_1, i_2, i)
Green	$(i_1 : 0, i_2 : \ 0, i : \ 0)$ $(i_1 : 1, i_2 : \ 1, i : \ 0)$
Dashed Blue	$(i_1 : 0, i_2 : \ 0, i : \ 1)$ $(i_1 : 1, i_2 : \ 1, i : \ 1)$
Red	$(i_1 : 1, i_2 : 0, i : 0)$ $(i_1 : 0, i_2 : 1, i : 0)$
Orange	$(i_1 : 1, i_2 : 0, i : 1)$ $(i_1 : 0, i_2 : 1, i : 1)$



P2 After considering the compositions in each part of problem **P1** (B)-(D), write down at least two safety properties for each of the parts in P1 (B)-(D) of the composed system that are true, and at least two safety properties that are violated by the composed system.

Solution: Examples of safety properties satisfied by all models in parts (B)-(D) are

(a) $\Box(x_1 \geq 0 \ \wedge x_2 \geq 0)$

(b) $\Box(x_2 \leq 3)$

Examples that are not satisfied by any of the three include

(a) $\Box(x_1 = x_2)$

(b) $\Box(x_2 > x_1)$