1. Suppose you construct a polynomial approximation to the function \( f(x) = \ln(x) \) on the interval \([1,3]\). If you use \( n \) equally spaced points, give an upper bound for the interpolation error using the error bound for equally spaced points given in class. How many points would be required to ensure the interpolation error is less than \( 10^{-7} \)?

2. (a) Use the polynomial interpolation feature of Matlab to interpolate the function \( f(t) = |t| \sin(t) \) at 21 equally spaced points on the interval \([-2,3]\) (spacing = 0.5). Remember to use a polynomial of degree 20.
(b) Plot the polynomial and the function \( f(t) \) together on the interval \([-2,3]\). Use a grid spacing of 0.01.
(c) About what is the largest value of the error \( |p(t) - f(t)| \) on that interval? At what point does it occur?
(d) Why would one expect this function to be hard to fit with a polynomial?

3. (a) Interpolate the data of the previous problem with a cubic spline using the spline interpolator of Matlab.
(b) Again, plot the spline \( s(t) \) and the function \( f(t) \) on the interval \([-2,3]\), with a grid spacing of 0.01.
(c) In addition plot the difference \( s(t) - f(t) \)
(d) What is the largest value of the error? Where does it occur?
(e) How does the spline interpolation compare to the polynomial?

Turn in your plots for this and the preceding problem.