Assignment 4
CSCI 3656
due February 10, 2011

1. Given that the distance in meters fallen from rest by a skydiver is

$$y(t) = \ln(\cosh(t\sqrt{gk}))/k.$$

compute the time taken to fall 85 meters using Newton's method. Here, gravitational acceleration $g = 9.8065$ m/s/s, and $k = .00341\text{m}^{-1}$ As an initial point use $t = 20$. Use a tolerance on the step size of 0.00001. Print the values of the time and the function at each iteration. Note that if y(t) is the distance fallen, then the derivative is given by

$$y'(t) = \sqrt{g/k}\,\frac{\sinh(t\sqrt{gk})}{\cosh(t\sqrt{gk})}.$$

(a) Observe the value of $|f(t_k)|$ at each iteration. Does Newton's method appear to have the quadratic convergence promised in theory?
(b) For the same problem, try Newton's method with starting point $t = 0.001$. Discuss your results, explaining why the method does not work as well

2. Try the secant method on the same problem. Use the starting points 20 and 21. Print out t and f at each iteration. How does the performance compare to Newton?

3. Consider the problem of finding the reciprocal of a real number a by solving the equation $f(x) = 1/x - a = 0$. Show how you can apply Newton's method to this problem, even if you are using a computing device that is not capable of performing division. Try this out for $a = 3.0$; take three steps starting at $x = 0.1$.

   For what set of initial values x_0 should we expect this iteration to converge? Try drawing a graph to understand this question.