

# Journal of Experimental Psychology

VOL. 59, NO. 2

FEBRUARY 1960

## EFFECT OF FIRST-ORDER CONDITIONAL PROBABILITY IN A TWO-CHOICE LEARNING SITUATION<sup>1</sup>

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The experiments reported here investigate behavior in a two-choice probabilistic task in which the independent variable is the conditional probability in the sequence of stimulus events. Each of the two stimulus events (lights) was required to occur equally often, but a partial patterning was introduced by varying the first-order conditional probability in the sequence of lights. This variable, denoted by  $\pi_{11}$ , is defined as the probability of occurrence of Light 1 on any trial, given that Light 1 occurred the previous trial. When  $\pi_{11} < .5$ , the lights tend to alternate on successive trials; when  $\pi_{11} > .5$ , each light tends to repeat itself. Thus,  $\pi_{11} = .7$  represents a 70% repetition pattern, and  $\pi_{11} = .3$  repre-

sents an equally strong alternation pattern. Since each light occurs equally often, the appropriate dependent variable is the frequency of *repetition responses*, a repetition response being defined as the response of predicting on the current trial that same *light* which occurred on the previous trial. An alternation response is analogously defined as predicting that light which did not occur on the previous trial.

This conditional probability variable was first used by Hake and Hyman (1953). It is of considerable interest because of its close relationship to the *frequency* variable for which the Estes and Straughan (1954) model predicts the "matching" behavior first reported by Grant, Hake, and Hornseth (1951). Engler (1958) tested a Bush-Mosteller (1955) model which predicts both the absolute frequencies of the two choices, and the level of repetition responses. The model succeeded with the first dependent variable, but failed to fit the repetition response results.

The first of the experiments reported here was designed to test a model (Anderson, 1956) for the con-

<sup>1</sup>Based on work submitted in partial fulfillment of the requirements for the PhD degree at the University of Wisconsin, 1956. The work was done while the author was an NSF predoctoral fellow. I wish to express my thanks to D. A. Grant, W. J. Brogden, and H. F. Harlow for their advice and helpful criticism. Thanks are also due S. H. Sternberg and R. C. Atkinson for a critical reading of the manuscript.

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ditional probability situation. The model is based on the ideas of Estes (1950) and of Restle (1955). It is assumed that there are two distinct subsets of stimulus elements, one of which is specific to the repetition response, the other specific to the alternation response. On any trial the sampled stimulus elements specific to a given response become conditioned to that response if it was indicated as correct, regardless of whether *S* actually made the response. However, if the given response was not the correct one, then the sampled stimulus elements specific to that response go into a neutral state. The probability of a repetition response on any trial is the ratio of the number of sample elements conditioned to the repetition response to the total number of sample elements in the conditioned state. Neutral elements have no effect on response probability, but they may be conditioned on later trials. (The model also postulated a third set of stimulus elements corresponding to responses other than the two above. It was assumed that such stimulus elements were progressively neutralized over trials. This third set of elements would thus affect the learning rate, but not the asymptotic level. Hence, it need not be further considered here.)

This model leads to the following approximate expression for the asymptotic probability of repetition responses:

$$\frac{1}{1 + c(1 - \pi_{11})/\pi_{11}} \quad [1]$$

where  $\pi_{11}$  is the conditional probability of Light 1, given that Light 1 occurred on the previous trial, and  $c$  is a constant which measures the relative weights of the two sets of stimulus elements.

Since the completion of the present

work, Burke and Estes (1957) have revised the original Estes and Straughan (1954) model, taking into account the nonindependence of the sample of active stimulus elements on successive trials. The revision, it may be noted, yields some small changes in the predicted asymptotes for the event frequency situation. More pertinently, it also yields the following expression for asymptotic repetition response level:

$$\pi_{11} + \theta\pi_{11}(1 - \pi_{11}) \quad [2]$$

where  $\theta$  is the learning rate. This model will also be tested against the present data.

Upon completion of Exp. I, it was seen that the model, although it gave a reasonable account of the asymptotic acquisition performance, was unable to handle the behavior in the transfer condition in which the various acquisition groups failed to converge to a common level despite the fact that they were receiving the same treatment. Since this differential transfer effect appeared to create difficulties for the mathematical approach in general, it seemed desirable to abandon the testing of specific models and begin a more extensive empirical investigation. Experiment II was accordingly designed to give a parametric treatment of the conditional probability variable. The data were recorded so as to permit the easiest tabulation of the sequential dependencies in the stimulus-response sequences. It was expected, both on theoretical grounds (Anderson, 1959) and from the results of earlier work (Estes & Straughan, 1954; Anderson & Grant: 1957, 1958; Jarvik, 1951), that these dependencies would give additional insight into the processes underlying the behavior in this situation.

## METHOD

The procedure common to the two experiments will be given first; that particular to the individual experiments will be found under the corresponding headings. Each group is labeled by 100 times its acquisition value of  $\pi_{11}$ . Thus, Group 30 received  $\pi_{11} = .3$  in acquisition, etc.

### Apparatus

The apparatus was the same as that used in previous work (Anderson & Grant, 1957). Two 6-w. opal lights, mounted 10 in. apart horizontally on a vertical, flat black, 15 × 20-in. panel, formed a stimulus display. The two lights were activated by a punched tape fed through a Western Union tape transmitter. An 800-cps tone served as the signal to respond. Presentation of tone and lights was automatic.

A row of four partially separated booths was placed 9 ft. from the panel of lights. Each booth contained a small box with two non-self-releasing keys, corresponding to the two lights. The *Ss* were required to push the keys down, and then push them up, in making their responses. Stimuli and responses were recorded automatically with an Esterline-Angus operation recorder.

Except for the panel of lights and the booths, all equipment was placed on *E*'s desk which lay behind the booths, and from which *E* monitored the experiment.

### Procedure

The *Ss* were volunteers from elementary psychology classes who received class points for their work. They were randomly assigned, in groups of two, three, or four, to the various experimental conditions.

At the beginning of the experiment, *Ss* were seated in the booths and read the instructions which were similar to Instructions C of Anderson and Grant (1957). In addition to operating detail concerning the keys, tone, and lights, the instructions stated that the task was to predict which of the two panel lights would come on next, that this was an experiment in learning, and that *Ss* should do the best they could even though the task seemed difficult. Three practice trials, in which both lights flashed each time, were then given, and questions answered. Finally, *Ss* were told that the rest of the experiment would have to be run off without conversation or interruption. The sequence of trials was continuous, with no external

indication of change from acquisition to transfer condition.

### Independent Variables

The two experimental variables were trial rate, and first-order conditional probability. The temporal division of each trial was as follows: signal tone (during which responses were to be made), 2.0 sec.; waiting time, 1.2 sec.; stimulus light on, .8 sec.; "intertrial interval," 1.0, 3.5, or 6.0 sec. These intertrial intervals thus yielded trial rates of 5.0, 7.5, and 10.0 sec. per trial.

First-order conditional probability has been defined in the introduction. The sequences of lights were random subject to the following rules: (a) exactly one light flash each trial; (b) the assigned value of  $\pi_{11}$  hold exactly within each block of 50 trials; (c) the absolute frequency of each light lie between 20 and 30 in each block of 50 trials. Four independent sequences were constructed for each acquisition value of  $\pi_{11}$ . In the transfer condition, four independent sequences were also used, but these were the same for all groups within each experiment.

*Experiment I.*—This experiment was based on a 3 × 2 design using the three trial rates, 5, 7.5, and 10 sec. per trial, and the two acquisition values of conditional probability,  $\pi_{11} = .3$ , and  $\pi_{11} = .7$ . Light sequences were kept orthogonal to these two variables. Each group received 200 acquisition trials. In the transfer condition,  $\pi_{11} = .5$  for all groups. The 5, 7.5, and 10 sec. groups were given 300, 200 and 100 transfer trials, respectively.

*Experiment II.*—This experiment was a parametric study of the effect of conditional probability on acquisition, on transfer to an extinction condition, and on recovery. A group of *Ss* was run at each of the 11 acquisition values of  $\pi_{11}$  ranging from 0 to 1 in steps of .1. The transfer condition was the same for all groups, with  $\pi_{11} = .5$  as in Exp. I. Acquisition and transfer were given in Session 1; recovery was tested in Session 2.

Trials were presented at the 5-sec. rate used in Exp. I. New stimulus sequences were constructed for conditions duplicating those of Exp. I. Instructions were reworded slightly by incorporating an additional statement to the effect that, while *S* should be able to do considerably better than just guessing, a perfect score was impossible, so that *S* should not worry if he was not correct every time.

Groups 10-90 were given 300 acquisition trials, followed by 200 transfer trials. Groups

0 and 100 received 100 acquisition trials, and 400 transfer trials. For these two groups, right and left were counterbalanced in Trials 1-100 within each of the four transfer sequences. Twenty Ss were run in each group, except that Group 50 contained 40 Ss.

Two additional subgroups, each with  $N = 12$ , were run with  $\pi_{11} = .3$ , and  $.7$ . The sequences for these groups were identical with those of the corresponding conditions of Exp. I for the first 200 trials, and with those of Exp. II for the remaining trials. These two groups permitted an evaluation of effect of the change in instructions from Exp. I to Exp. II, without confounding any possible effect of sequences. Since no effect of instructions was found, these Ss have been combined with the others so that Groups 30 and 70 each have  $N = 32$ .

Recovery was tested by giving 60 additional trials with  $\pi_{11} = .5$ . Fifteen such sequences were constructed by taking successive blocks of 60 trials from three of the transfer sequences of Exp. I. The Ss were assigned to these sequences unsystematically, but each sequence was used about equally often. Session 2 was run after an interval of two to six days. An S was signed up for Session 2 before serving in Session 1, according to E's schedule and each S's own convenience. In contrast to Session 1, Ss were occasionally run alone in Session 2.

One S adopted the deliberate policy of leaving his key down for several trials until he wished to change his choice of responses. He was replaced although his record seemed normal in other respects. Three Ss, all from the additional subgroup run under the  $.7$  condition, did not return for Session 2. The Session 2 record of one S was lost because of a recording error.

## RESULTS

A repetition response is defined as a press of that key corresponding to the light which flashed the previous trial. Since two stimuli and two responses are involved, a nonrepetition response may be called an alternation response. For conditions with  $\pi_{11} > .5$ , a repetition response is also an optimal response since it is more likely to be correct. Similarly, when  $\pi_{11} < .5$ , an alternation response is an optimal response. The main de-

pendent variable is repetition response frequency.<sup>3</sup>

Unless otherwise noted, the analysis of variance was used in the statistical analyses. The Sequence variable, corresponding to the different random sequences used, was treated throughout as a fixed variate (Wilk & Kempthorne, 1955, p. 1163).

### *Mean Performance Curves*

*Experiment I.*—The results of Exp. I are shown in Fig. 1, which plots mean percentage of repetition responses as a function of trial blocks. Acquisition is rapid, but marked reversals occur in the third block of 10 trials for Groups 30 (alternation groups), and in the fifth block of 10 trials for Groups 70 (repetition groups). Terminal response levels, calculated over Trials 151-200, average 79% for Groups 70, and 33% for Groups 30. These values correspond to optimal response levels of 79% and 67%, respectively. No trial rate effect is apparent for Groups 70, but the slower rate conditions of Groups 30 do somewhat poorer in the middle 100 acquisition trials. There is an immediate change in the transfer trials, in which all groups received the same purely random sequences. However, Groups 30 and 70 show little sign of approaching a common final level, even after the 300 transfer trials given the two 5-sec. groups.

Analyses of acquisition performance on total responses over Trials 1-200,

<sup>3</sup> Between .003 and .004 of the responses were "improper," with S pressing too late, pressing neither key, or, most frequently, pressing both keys. Frequency of such improper responses did not appear to depend on trial rate, conditional probability, or trial number, except that Groups 0 and 100 made no improper responses in acquisition. All improper responses were filled in randomly in order to simplify the tabulations.

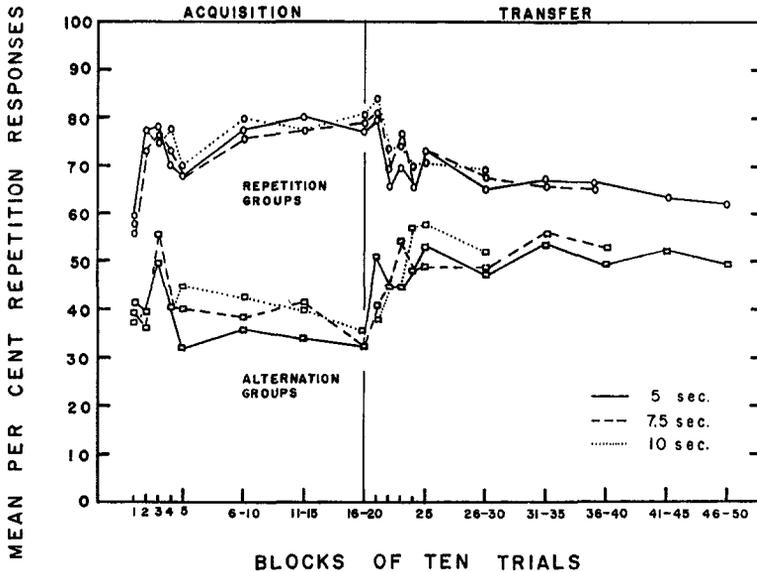


FIG. 1. Mean percentage of repetition responses in acquisition and in transfer as a function of trial blocks, Exp. I.

and over Trials 151–200 were performed for Groups 30 and Groups 70 separately. There were no significant results in the analyses of Groups 70, although the Sequence effect approached the .05 level in the first analysis. For Groups 30, the Sequence effect was significant in both analyses:  $F(df = 3, 48) = 5.46$ , and  $4.40$ , for the analyses over Trials 1–200, and Trials 151–200, respectively. No other reliable differences among Groups 30 were found, although the linear component of trial rate approached significance,  $F(df = 1, 48) = 3.82$  in the analysis of the total score over Trials 1–200. The comparison of optimal response frequencies, based on the pooled error term, yielded significant differences between the two conditional probability conditions:  $F(df = 1, 96) = 97.7$  and  $51.1$ , for the tests over Trials 1–200 and Trials 151–200, respectively.

Analyses of transfer performance

were made on repetition response frequencies, and included Sequences as an additional factor orthogonal to the two main independent variables. The effect of the conditional probability factor was significant in all analyses. In particular, total repetition responses over Trials 451–500 for the two 5-sec. groups were significantly different,  $F(df = 1, 32) = 5.15$ . No other main effect or interaction approached significance.

The first 50 trials of the sequences of reinforcing lights were inspected for peculiarities which might explain the reversals early in acquisition. The reversal for Groups 70 apparently arose entirely from a run of seven consecutive alternations which occurred in one of the sequences. Nothing unusual was seen in the sequences for Groups 30, however. See also below.

*Experiment II.*—Figure 2 plots the mean percentage of repetition responses as a function of trial blocks

with acquisition value of  $\pi_{11}$  as a parameter. Following the rapid acquisition, the various groups level off in the expected order with some conditions showing further slow changes. Three conditions deserve special mention. Group 30 exhibits an anomalous reversal similar to that seen in Exp. I. Group 40 does well in terms of optimal responses on Trials 1-10, but then ascends to the "chance" level where it remains for the duration of Session 1. Group 50 stabilizes at a repetition response level of 62% although the sequences of lights were entirely random in this condition. This tendency toward repetition responses in Group 50 is consistent with the finding of Exp. I that Groups 70 made more optimal responses than Groups 30. This latter result appears again in Exp. II, and the same trend is shown in the comparisons of Groups 40 and 60, and of Groups 20 and 80.

Table 1 gives the means and *SEs* obtained from total repetition response frequencies over Trials 201-300. This table includes the theo-

retical asymptotic response levels predicted by the author's model, and by the Burke and Estes (1957) model. The single unknown parameter in each model was evaluated from the data for Group 50 by equating obtained and theoretical asymptotes for this condition. This procedure yielded  $c = .618$  for the present model, and  $\theta = .472$  for that of Burke and Estes. The two models make essentially the same predictions, and these predictions are quite good for the larger values of  $\pi_{11}$ . It is clear, however, that neither model gives an adequate account at the lower values of  $\pi_{11}$ , where the groups show an increasing tendency to lie below the theoretical asymptote.

As is evident in Fig. 2, Groups 0 and 100 were shifted to the transfer condition after only 100 acquisition trials. The main results for these two conditions will be given in this paragraph and they will not be considered again unless explicitly mentioned. Mean errors in acquisition were 1.35 and 2.95 for Groups 0 and 100, respectively. The difference is significant as shown by the Mann-Whitney *U* test,  $z = 2.93$ . The groups overshoot each other in the fourth block of transfer trials, and Group 0 makes significantly more repetition responses in that block,  $F(df = 1, 32) = 6.59$  by an ad hoc test. The response rate over Trials 201-500 was quite stable, but these data are omitted in Fig. 2. Group 0 averaged 3.7% fewer repetition responses than Group 50, and Group 100 averaged 2.3% more repetition responses than Group 50 over these last 300 trials. However, the between-groups *F* for these data was not significant.

In the transfer trials, all conditions show some initial change in response rate except for Groups 40, 50, and

TABLE 1

OBTAINED AND PREDICTED MEAN PROPORTION OF REPETITION RESPONSES ON TRIALS 201-300, EXP. II

Group	Obtained	Predicted		<i>s<sub>m</sub></i>
		Eq. 1	Eq. 2	
90	.94	.94	.94	.011
80	.90	.87	.88	.014
70	.84	.79	.80	.014
	(.77) (.79) (.80)			
60	.75	.71	.71	.020
50	.62	.62	.62	.021
40	.49	.52	.51	.025
30	.35	.41	.40	.014
	(.33) (.33) (.35)			
20	.16	.29	.28	.020
10	.04	.15	.14	.007

Note.—Entries in parentheses are from Trials 151-200 of Exp. I.

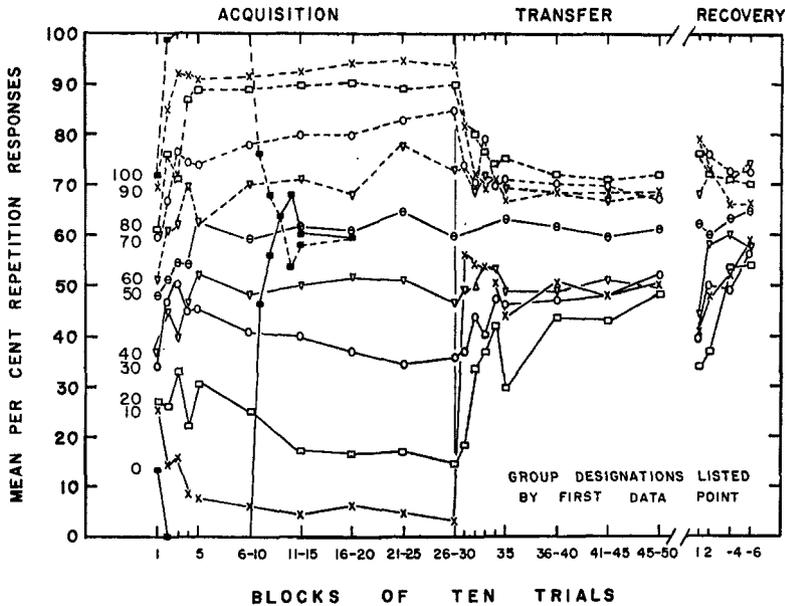


FIG. 2. Mean percentage of repetition responses in acquisition, in transfer, and in recovery, as a function of trial blocks, Exp. II.

60. Group 10 exhibits a sharp increase comparable to Group 0. The terminal transfer levels appear to be undergoing little change. However, these terminal levels are not smoothly related to acquisition treatment: Groups 60-90 all cluster around the 70% level, Groups 10-40 level off at about 50%, and Group 50 falls neatly between.

An analysis of total repetition responses over Trials 401-500 was performed with sequences of reinforcing lights and acquisition treatment as orthogonal factors. For acquisition treatment,  $F(df = 8, 188) = 16.8$ , but no other effects were significant. Using the value  $s_m = 2.64$  (for a group of 20 Ss) obtained from this analysis in Duncan's (1955) range test, Groups 10-40, Group 50, and Groups 60-90 separated out as three significantly different subsets of conditions, with no further differentiation within these three subsets.

Recovery is evident for most groups in Fig. 2, but dissipates in about 20 trials. Although Session 2 was too short to be more than suggestive, there seems to be a tendency for Groups 60-90 to return to their final level in Session 1, and for Groups 10-40 to rise above their final level in Session 1.

Recovery scores were computed by taking the absolute value of the number of repetition responses on Trials 481-500 from twice the number of repetition responses on Trials 1-10 of Session 2. A positive score thus indicates a recovery of the acquisition response. Groups 0, 50, and 100 were tested together but the analysis yielded no significant results. The mean recovery score, averaged over the remaining eight groups, was significantly different from zero,  $F(df = 1, 173) = 32.3$ . Further tests are vitiated by the lack of control over the interval between sessions.

Except for Group 100, right and left were not counterbalanced in the light sequences because position preferences cannot produce artifacts with the present independent variable. Position preferences may exist, of course, and two tests were made for them. First, all key presses for all Ss over Trials 451-500 were pooled. There was a slight excess of left key presses, but a binominal test failed of significance. Second, all repetition responses of Group 50 over Trials 1-50, and over Trials 1-500 were treated similarly. No significant positional effects were found in these two tests either.

### *Sequential Dependencies*

Considerable additional information may be extracted from the data by considering the dependence of the response on any trial upon the subsequence of stimuli and responses on the trials immediately preceding. The analysis of these sequential dependencies has proved useful in previous work on specific models (Anderson & Grant: 1957, 1958; Estes & Straughan, 1954). Without a specific model, however, the interpretation of the sequential dependencies presents certain pitfalls which have given rise to some confusion in the literature. Accordingly, some comments will be made at appropriate points on limitations in the interpretation of such data.

The two stimulus events will be denoted by 0 and 1. A subsequence of consecutive events will be called a *tuple*. The symbol,  $R'(T)$ , will denote the proportion of repetition responses conditional on the occurrence of any particular tuple,  $T$ . Thus,  $R'(100)$  denotes the proportion of repetition responses, given that the stimulus lights on the three preceding trials were 1, followed by 0, followed by 0. Its value was calculated by dividing the number of occurrences of the stimulus tuple, 100, into the number of times that this tuple is followed by a repetition response

which, in this case, would be the response of predicting Event 0. Because right and left were equiprobable in all light sequences, and because no position preferences were found, the data from each pair of positionally complementary tuples were pooled before dividing to obtain the conditional proportions. Consequently,  $R'(100)$  and  $R'(011)$  denote the same quantity, and this quantity is obtained by pooling the data from the two 3-tuples, 100, 011.

The specific tabulational procedure used here requires comment in two respects. In the first place, the compilations were done by pooling the data over all Ss and all sequences in each condition before computing the desired proportions. There can result, as a consequence, certain biases as discussed more fully below. Second, the method of tabulation was such that, for instance, the tuple, 0000, contributed one instance to  $R'(0000)$ , two instances to  $R'(000)$ , three instances to  $R'(00)$ , and four instances to the repetition response rate,  $R'(0)$ . Similarly, the alternation tuple, 1010, contributed one instance to  $R'(1010)$ , two instances to  $R'(010)$ , three instances to  $R'(10)$ , and four instances to the repetition response rate,  $R'(0)$ ; (see penultimate sentence of preceding paragraph). This procedure averages the  $R'$  values over all possible antecedents of the corresponding tuples, and thus places the  $R'$  values for runs, alternations, and other tuples on an equal footing. A somewhat different tabulational procedure has been used by Nicks (1959) in that part of his report dealing with run curves. The present method leads to simpler mathematical expressions relating sequential dependencies and model parameters. However, Nicks' method may prove to be more useful for nonparametric analyses.

It may be worth mentioning that these tabulations, especially those involving response dependencies, are time consuming. Consequently it is advisable to plan the general tabulation procedures and the data recording sheets with care so as to maximize efficiency. The use of the symbols 0 and 1, rather than

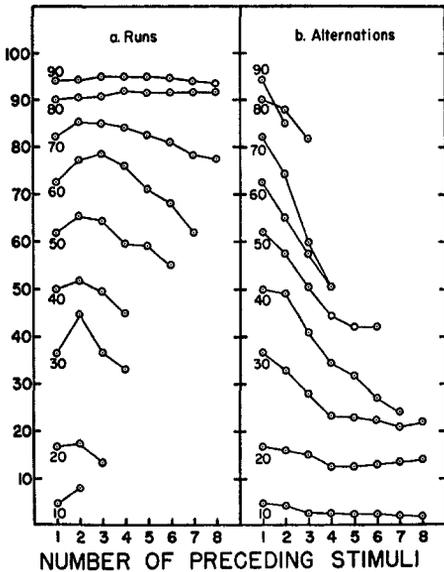


FIG. 3. Mean percentage of repetition responses over Trials 101-300, inclusive: (a) response following runs of the designated length; (b) response following alternation tuples of the designated length. See text.

R and L, is recommended since they are easier to write and to read.

*Responses following runs and alternation tuples.*—The proportion of repetition responses following runs and alternations tuples of various lengths were tabulated from the data of Trials 101-300 for each group. The results are given in Fig. 3. Tuples of length greater than eight are omitted, as well as all data points based on fewer than 100 instances in the denominator. Figure 3a plots the data for runs of various lengths. The successive points on each curve give the values of  $R'(0)$ ,  $R'(00)$ ,  $R'(000)$ ,  $R'(0000)$ , etc. The inverted bowl shape of the five middle curves is the well known Jarvik (1951) negative recency effect, or gambler's fallacy. In Fig. 3b are shown the data for the alternation tuples. The successive points on each curve give the values of  $R'(0)$ ,  $R'(10)$ ,  $R'(010)$ ,  $R'(1010)$ , etc. As would be expected, the probability of a repetition response decreases as the length of the alternation tuple increases. Some-

what more striking is the fact that in all groups (except Group 90 for which the relevant datum does not appear) the decrement following the third light in an alternation tuple is greater than the decrement following the second light in such a tuple. This result is not an artifact of the tabulational procedure. Indeed, if each point included data only from those alternation tuples having *precisely* the length specified on the abscissa, an even greater difference in the decrements would have been obtained. Finally, by comparing the two panels of Fig. 3, it can be seen that an alternation tuple has a greater overt effect than does a run.

The variation over trial blocks in the response following runs and alternation tuples is exhibited in Tables 2 and 3. These tables list the differences between the  $R'$  values for tuples of lengths 2, 3, and 4, and the  $R'$  values for tuples of the next shorter length. The values of  $R'(0)$  are included in Table 2 in order to permit calculation of the separate  $R'$  values entering into the several differences.

A negative difference in Table 2 indicates that the probability of a repetition response decreases as the length of a run increases, i.e., a negative recency effect. The lowest order differences,  $R'(00) - R'(0)$ , are negative over the first 50 trials, but thereafter are nearly all positive. The two higher order differences also increase over trial blocks. These results suggest that the negative recency effect adapts out over trials. This adaptation seems to be more rapid for conditions with stronger repetition tendencies in the light sequences.

A positive difference in Table 3 indicates that repetition response probability decreases as the length of the alternation tuple increases. Most of the values of  $R'(0) - R'(10)$  are negative in the first block of 50 trials. Thereafter they assume fairly stable positive values except perhaps in the transfer trials for Groups 10-30. The values of  $R'(10) - R'(010)$  are fairly constant over trial blocks. These latter values are also uniformly larger than the values of  $R'(0) - R'(10)$ ,

TABLE 2  
DIFFERENCES OF  $R'$  VALUES FOR RUNS OF VARIOUS LENGTHS

Trial Block	$R'$ and $R'$ Diff.	Group								
		10	20	30	40	50	60	70	80	90
1-50	$R'(0)$	13	27	45	45	55	62	72	79	89
	$R'(00) - R'(0)$	01	-04	-03	-09	-04	-08	02	-02	-01
	$R'(000) - R'(00)$			-08	-12	-08	-08	-02	00	01
51-100	$R'(0)$	06	25	41	48	59	70	78	89	91
	$R'(00) - R'(0)$	01	04	01	01	02	04	06	02	02
	$R'(000) - R'(00)$			-06	-13	-07	-02	00	00	00
	$R'(0000) - R'(000)$			(-05) <sup>a</sup>		(-03) <sup>a</sup>		(-02) <sup>a</sup>		
101-200	$R'(0)$	05	17	38	51	61	70	80	90	93
	$R'(00) - R'(0)$	02	00	07	03	06	04	04	01	01
	$R'(000) - R'(00)$			-09	-06	00	00	-01	00	00
	$R'(0000) - R'(000)$			-02		-03		-02		
201-300	$R'(0)$	04	16	35	49	62	75	84	90	94
	$R'(00) - R'(0)$	05	01	09	00	02	05	03	00	01
	$R'(000) - R'(00)$			-07	02	-02	03	00	01	01
	$R'(0000) - R'(000)$			-03		-07		00		
301-400	$R'(0)$	51	39	45	49	62	69	72	74	70
	$R'(00) - R'(0)$	-04	04	08	02	05	06	07	05	06
	$R'(000) - R'(00)$	03	-07	-01	01	-02	04	00	01	03
	$R'(0000) - R'(000)$			-02		-02		-02		
401-500	$R'(0)$	49	46	50	50	60	68	68	71	69
	$R'(00) - R'(0)$	-01	05	07	01	02	05	06	01	07
	$R'(000) - R'(00)$	00	-03	-02	-01	00	05	03	04	03
	$R'(0000) - R'(000)$			-03		01		-01		

Note.—Decimal points omitted.

<sup>a</sup> Calculated from the data of Trials 1-100.

confirming and extending the results of Fig. 3b.

*Response following other stimulus tuples.*

—It is also of interest to examine the sequential dependencies for stimulus tuples other than runs and alternations. The statistic considered is the difference of  $R'$  values of tuples which are alike except for the leftmost (trialwise most remote) stimulus event. These differences thus measure the influence of the trialwise most remote event of the tuple on the current response.

The  $R'$  values have unequal reliability since the number of instances of a given tuple varies systematically with acquisition treatment. The  $R'$  values presented for 3-tuples are based on at least 150 instances in a block of 100 trials. However, six of the 4-tuples of Table 5 had pooled frequencies between

50 and 100 instances. In the transfer trials, of course, all tuples of a given length are equiprobable, and the  $R'$  values of tuples of lengths 2, 3, and 4 are based on approximately 1000, 500, and 250 instances, respectively, for a group of 20 Ss. It should be noted that pooling data across the sequences within each condition introduces a bias to the extent that the number of instances of each tuple varies across sequences. However, the bias would appear to be negligible in the present results. Of the 24  $R'$  values which were also computed from the individual S's data (see below), the average magnitude discrepancy was .005.

The 2-tuple difference,  $R'(00) - R'(10)$ , may be obtained by adding corresponding values of  $R'(00) - R'(0)$  from Table 2 and  $R'(0) - R'(10)$  from Table 3.

TABLE 3  
DIFFERENCES OF  $R'$  VALUES FOR ALTERNATION TUPLES OF VARIOUS LENGTHS

Trial Block	$R'$ Differences	Group								
		10	20	30	40	50	60	70	80	90
1-50	$R'(0) - R'(10)$	00	-01	-02	-06	-02	-12	05	-03	-01
	$R'(10) - R'(010)$	01	02	06	09	04	25	20		
51-100	$R'(0) - R'(10)$	00	01	00	01	02	06	15	07	10
	$R'(10) - R'(010)$	03	01	08	05	10	14	17		
	$R'(010) - R'(1010)$			(04) <sup>a</sup>		(05) <sup>a</sup>		(11) <sup>a</sup>		
101-200	$R'(0) - R'(10)$	00	00	03	02	06	08	10	04	08
	$R'(10) - R'(010)$	02	02	05	09	07	08	11		
	$R'(010) - R'(1010)$			04		06		05		
201-300	$R'(0) - R'(10)$	01	01	04	00	02	07	06	00	09
	$R'(10) - R'(010)$	01	00	05	07	07	07	18		
	$R'(010) - R'(1010)$			05		06		13		
301-400	$R'(0) - R'(10)$	-04	04	08	02	04	06	08	04	06
	$R'(10) - R'(010)$	01	10	10	10	10	09	13	13	17
	$R'(010) - R'(1010)$			05		07		10		
401-500	$R'(0) - R'(10)$	-02	05	06	00	01	06	06	01	08
	$R'(10) - R'(010)$	01	10	11	12	06	10	11	19	13
	$R'(010) - R'(1010)$			06		02		11		

Note.—Decimal points omitted.

<sup>a</sup> Calculated from data of Trials 1-100.

TABLE 4  
DIFFERENCES OF  $R'$  VALUES OF PAIRED 3-TUPLES

Trial Block	Group									
	10	20	30	40	50	60	70	80	90	
(a) $R'(000) - R'(100)$										
1-50			-11	-19	-17	-17	-08	-02	06	
51-100			-08	-21	-16	-05	00	01	03	
101-200			-17	-09	-01	-02	-04	01	00	
201-300			-09	03	-04	07	00	04	04	
301-400	07	-13	-02	01	-03	07	00	03	06	
401-500	01	-05	-03	-01	01	09	06	07	06	
(b) $R'(110) - R'(010)$										
1-50	12	08	18	20	09	35	30			
51-100	22	03	26	13	22	25	25			
101-200	15	06	19	19	13	13	16			
201-300	08	03	15	15	13	11	23			
301-400	01	18	18	19	18	17	24	24	30	
401-500	01	18	21	21	11	19	20	18	24	

Note.—Decimal points omitted.

These differences rise from predominantly negative initial values to level off at reasonably stable and definitely positive values of the order of .1. Except for Groups 10 and 20, there seems to be neither any systematic between-groups variation, nor any change in the transfer trials.

The two 3-tuple differences are presented in Table 4 for each group and for successive blocks of trials. The separate  $R'$  values in each difference may be readily evaluated by making use of the data of Tables 2 and 3. The values of  $R'(000) - R'(100)$  are mostly negative in the early trials, but tend to become positive in the later trials for the groups with higher acquisition values of conditional probability. This trend presumably reflects the adaptation of the negative recency effect previously mentioned. The values of  $R'(110) - R'(010)$ , although suggestive of some differences between groups and over trial blocks, are rather the more remarkable for the degree of constancy which they exhibit. The average value is of the order of .2 which, being larger than the value of  $R'(00) - R'(10)$ , re-emphasizes the influence of alternation tuples on  $S$ 's response.

In order to obtain some indication of the variability of the scores discussed in the preceding two paragraphs,  $R'$  values were computed for the individual  $S$ s of Groups 30, 50, and 70, for Trial Blocks 101-200, and 401-500. The resulting analyses are given in Table 5.

In particular, the  $F$ s for Mean show that the  $R'$  values differences, averaged over the two blocks of trials and over groups, are significantly different from zero.

The four 4-tuple comparisons were made only for Groups 30, 50, and 70; the results are given in Table 6. The values of  $R'(0000) - R'(1000)$  are largely negative, again reflecting the negative recency effect. The values of  $R'(0010) - R'(1010)$  are all positive and fairly large. Thus an alternation rather than a repetition three trials back in the sequence of lights still exerts a decided effect on the current response. The two remaining differences are essentially zero for Groups 30 and 50, but tend to be positive for Group 70.

The results of this section give ample evidence of the influence of the more remote stimulus events on the current response. In interpreting these results, however, it should be specifically noted that a nonzero difference implies nothing about  $S$ 's perception of, or memory for these more remote stimuli. Any model which takes account of the trial-to-trial changes in the response probability would predict nonzero differences (Anderson, 1959; Estes & Straughan, 1954).

*Response-response dependencies.*—Further information may be obtained by considering the dependence of the current response on the preceding response. Unfortunately, these dependencies are more difficult to interpret than are the stimulus dependencies because of a selection effect, and because of certain

TABLE 5  
ANALYSIS OF VARIANCE OF  $R'$  VALUE DIFFERENCES FOR GROUPS 30, 50, AND 70

Source	df	$F$ Ratios For		
		$R'(00) - R'(10)$	$R'(000) - R'(100)$	$R'(110) - R'(010)$
Mean	1	51.9*	4.81*	80.1*
Groups	2	1.18	5.42*	1.80
Error (b)	101	(.045) <sup>a</sup>	(.041) <sup>a</sup>	(.069) <sup>a</sup>
Trials	1	2.75	8.56*	0.23
T × G	2	3.40*	2.59	0.66
Error (w)	101	(.020) <sup>a</sup>	(.036) <sup>a</sup>	(.032) <sup>a</sup>

\*  $P < .05$ .

<sup>a</sup> Error mean squares.

TABLE 6  
DIFFERENCES OF  $R'$  VALUES OF PAIRED 4-TUPLES

Trial Block	Group			Group		
	30	50	70	30	50	70
	(a)	$R'(0000) - R'(1000)$		(b)	$R'(1110) - R'(0110)$	
1-100	08	-10	-07	05	00	04
101-200	-03	-06	-07	-02	01	12
201-300	-06	-12	00	00	03	05
301-400	-04	-04	04	-02	-05	09
401-500	-07	01	-02	01	00	06
	(c)	$R'(0010) - R'(1010)$		(d)	$R'(1100) - R'(0100)$	
1-100	10	11	17	07	-03	07
101-200	12	11	06	00	01	04
201-300	16	10	17	01	02	02
301-400	11	15	20	00	-01	03
401-500	11	03	20	-06	-03	-03

Note.—Decimal points omitted.

biases. These two difficulties will be discussed in the appropriate places.

The simple first-order response-response dependency is found by counting the frequency with which the key press on one trial is repeated on the next trial. This statistic was computed for each  $S$  of Groups 30, 50, and 70, over Trials 401-500, with resulting mean proportions of .538, .533, and .538, respectively. The three values do not differ significantly among themselves, but their mean is greater than the chance value of .500,  $F(df = 1, 101) = 17.6$ . This same statistic was also calculated for each of the ten 50-trial blocks for Group 50, but there was very little trial variation. The mean over all 500 trials was .538.

The result of the previous paragraph does not necessarily imply either that  $S$ s had a set or tendency to repeat the previous key press, or that the previous response has any determining influence on the present response; it could equally well represent the workings of the laws of probability according to the following argument. The probability of a left key press, averaged over trials on which a left key press occurred, will be greater than this same quantity averaged over trials on which a right key press occurred. It may then be expected (at least in the

present case; see also below) that the probability of a left key press, averaged over trials following a left key press, will be greater than this same quantity averaged over trials following a right key press. The response will thus tend to repeat itself so that there is no present reason to believe that the obtained value of the response-response dependency represents anything more than this response selection effect. Any mathematical model would, of course, predict the magnitude of this response selection effect and thereby make use of the obtained data as a test of goodness of the model (e.g., Anderson, 1959; Anderson & Grant, 1958). It should be noted that, in general, some response selection effect will always be obtained whenever successive responses are not, for whatever reason, statistically independent. Consequently, the joint dependencies to be considered next are subject to the same difficulties of interpretation.

The dependence of the current response on the joint occurrence of the previous response and the four previous stimuli was also investigated. The notation for these dependencies is similar to that used for the stimulus dependencies. In the argument of each  $R'$  value, the symbols preceding the semicolon denote

the previous stimuli in the same way as before, and the symbol following the semicolon denotes the previous response. Thus,  $R'(1010; 1)$  denotes the proportion of repetition responses given that the last four stimulus lights were 1,0,1,0, in that order, and that the last response was 1. In each case, the last listed stimulus is the reinforcing stimulus for the listed response. As before, the data were pooled over key positions so that, for instance,  $R'(1010; 1)$  and  $R'(0101; 0)$  denote the same quantity. There is thus no restriction involved in presenting the data in such a way that the last stimulus is always 0, and this convention has been adopted. Consequently, the  $R'$  values always denote the conditional probability of the response 0 on the next trial.

These  $R'$  values were obtained by pooling the data over  $S$ s and over trials for Trial Blocks 1-300 and 301-500. The comparisons presented in Table 7 are differences between  $R'$  values based on the same stimulus 4-tuple, and differing only in the previous response. Nonzero differences correspond, therefore, to predictability conferred by knowledge of the previous response over and above predictability gained by knowledge of the four previous stimuli. Table 7 is so arranged that the last response was correct in the  $R'$  value

on the left side, incorrect in the  $R'$  value on the right side of each difference. In addition, the first four lines are for those cases in which the last two stimuli were alike, and the last four lines are for those cases in which the last two stimuli were unlike.

The problem of bias must be considered before discussing the results. Since the  $R'$  values are proportions conditional on the previous response, they involve a denominator which is subject to variation. This random variation introduces a bias which is approximately inversely proportional to sample size (Cramer, 1946, Sec. 27.7). An  $R'$  value computed for an individual  $S$  will thus be biased because of the limited number of responses made by that  $S$ . If  $S$ s were identical in the sense of having the same learning rates, this bias could (and should) be reduced by pooling the data of the group in order to compute a single ratio. However, if  $S$ s are not identical, this pooling procedure will itself introduce a bias because of the limited number of  $S$ s. This bias will be found even when there is sufficient data for each  $S$  to render negligible the bias in the individual ratios. In fact, the bias in the individual scores reported below is negligible; however, the entries of Table 7, which are based on pooled group data, do involve appreciable bias.

TABLE 7  
DIFFERENCES OF PAIRED  $R'$  VALUES FOR STIMULUS-RESPONSE DEPENDENCIES

$R'$ Differences	Group 30		Group 50		Group 70	
	Trials		Trials		Trials	
	1-300	301-500	1-300	301-500	1-300	301-500
$R'(0000; 0) - R'(0000; 1)$	51	37	37	34	34	35
$R'(1000; 0) - R'(1000; 1)$	21	26	20	15	22	22
$R'(0100; 0) - R'(0100; 1)$	04	09	13	21	15	12
$R'(1100; 0) - R'(1100; 1)$	07	10	10	21	11	04
$R'(1110; 0) - R'(1110; 1)$	-02	-02	-03	02	04	00
$R'(0110; 0) - R'(0110; 1)$	-05	04	-07	-07	-06	-01
$R'(0010; 0) - R'(0010; 1)$	-04	00	-06	-06	05	18
$R'(1010; 0) - R'(1010; 1)$	-07	01	-09	-04	-02	-11

Note.—Decimal points omitted.

This observed bias may, therefore, be taken as reflecting the existence of individual differences.

It may be noted, parenthetically, that any model which took into account the trial-to-trial changes in response probability would yield theoretical expressions (possibly of a Monte Carlo variety) for the *unconditional* proportions. The use of such data would avoid the problem of bias arising from the use of ratios with a random variable denominator. Since the two models considered here have already been seen to be inadequate, this procedure has not been followed. Without a model, however, the unconditional proportions are uninterpretable. Consequently, the nonparametric procedure adopted here, despite its defects, seems necessary to obtain meaning from the results.

The results for Group 50 will be considered first since they are the most reliable. The entries in the first four lines of Table 7 are positive and large. Positivity would be expected on the basis of the response selection effects discussed above since the previous key press in the  $R'$  values on the left of each difference is a 0, and since the repetition response is a 0 for both  $R'$  values of each difference. However, the Table 7 entries seem considerably larger than one would expect if the response selection effect alone were operating. The most appropriate measure of the size of the response selection effect is obtained by doubling the difference between the simple first-order response-response dependency and the chance level. The previous data of this section thus yield the value of .076 against which to compare the Table 7 entries. This comparison value may actually be too large since taking into account the four previous stimuli, as is being done here, would be expected to reduce the amount of information conferred by knowledge of the previous response.

In order to test this comparison, the bias in the entries of Table 7 must be allowed for. Two values of  $R'(000; 0) - R'(000; 1)$  were calculated for each  $S$  of Group 50, each value based on the

data of alternate 50 trial blocks. Since the bias is inversely proportional to the amount of data, comparing the mean of these two values with the corresponding value based on all the data for that  $S$  yields a measure of the bias in the individual scores arising from the use of the ratio. This bias was negligible. It may be noted, incidentally, that the correlation between the two scores was .55. The mean of the individual values of  $R'(000; 0) - R'(000; 1)$  was .24 which is significantly larger than the comparison figure of .076,  $F(df = 1, 39) = 50.83$ . Also, the .24 value is less than the weighted mean of .27 obtained from the Group 50 entries in the first two lines of Table 7. It is thus seen that these particular entries are biased away from zero.

The finding that the response selection effect is insufficient to account for the Group 50 entries in the upper half of Table 7 should be interpreted cautiously. It may be necessary to allow for a determining effect of the previous response on the present response as is done in the experimenter-subject controlled models of Bush and Mosteller (1955). A second possibility is that the assumption of path independence must be abandoned as in the models proposed by Sternberg (1959b).

The Group 50 entries in the lower half of Table 7 are predominantly negative. Statistical significance was investigated by computing individual values of  $R'(10; 0) - R'(10; 1)$  according to the procedure used for the upper half of the table. The split-half analysis yielded a correlation of .67, and showed also that the bias in the individual scores was negligible. The mean of the individual scores was  $-.036$ , with a standard error of .030. The tabular entries are again biased away from zero, since the corresponding mean obtained from Table 7 was  $-.053$ . Although the Group 50 entries in the lower half of Table 7 are not significantly less than zero, they deserve comment since the response selection effect noted above would, if taken at face value, imply that these entries should be somewhat greater than

zero. In accord with the original model, and with the preceding analysis of stimulus dependencies, it is assumed that Ss have an alternation response (predicting next that light which did not flash last) in their repertoire. Now, in the  $R'$  value on the left of each difference, an alternation response (a 0) has just been made (and rewarded). But in the  $R'$  value on the right side of each difference, a repetition response has just been made (and punished). Hence it would be expected on purely probabilistic grounds that an alternation response (a 1) would be more likely on the next trial in the first case than in the second. This means that a repetition response on the next trial will be less likely in the first case than in the second, in agreement with the data. By thus interpreting the overt key presses as alternation and repetition responses, it is seen that the simplest explanation of the negative values is in terms of a response selection effect. Conversely, the fact that the entries are not positive gives added support to the existence of the alternation response. The further interpretation of the results as indicative of differential effects of reward and nonreward would be premature.

The data for Groups 30 and 70 need little discussion. The entries in the upper half of Table 7 present essentially the same picture as Group 50 did. The data for the lower half of the table do not show such good agreement. This might perhaps be expected for Group 70 since the alternation response presumably does not develop as strongly there. For Group 30, individual values of  $R'(10; 0) - R'(10; 1)$  over Trials 1-300 were computed using the split-half technique as before. The correlation of the two scores was .69, and again the bias in the individual scores was negligible. The mean individual score was  $-.061$  with a standard error of .031. While the mean thus falls slightly short of being significantly less than zero, it reinforces the corresponding analysis of the Group 50 data. It should be noted, however, that the values over Trials 301-500 for Group 30 are not negative,

a result which is contrary to the above explanation.

### Miscellaneous Results

This section gives two results from the combined data of Exp. I and II.

It has been observed that Groups 30 in both experiments show a hump at the third block of 10 acquisition trials, first increasing and then decreasing in frequency of repetition responses. Tests were made on the difference of the total scores on Trials 1-10, and Trials 21-30. The mean difference was significantly greater than zero both for Exp. I,  $F(df = 1, 48) = 18.81$ , and for Exp. II,  $F(df = 1, 24) = 12.46$ . Moreover, there were no complications from other factors: in Exp. I, neither trial rate, stimulus sequences, nor their interaction were significant; in Exp. II, where eight different sequences were used, the  $F$  for Sequences was again close to unity. Group 40 lay significantly below the 50% level on the first block of 10 trials, which is consistent with the behavior of Group 30.

These results suggest the presence of an initial tendency toward alternation responses. In order to obtain a purer measure of initial tendencies, the data from the first two key presses are given in Table 8. Here the first stimulus light follows the first key press and precedes the second key press. Table 8 includes the present data, that from Exp. 3 of

TABLE 8  
JOINT DISTRIBUTION OF FIRST TWO KEY  
PRESSES AND FIRST REINFORCING  
STIMULUS

First Two Key Presses	First Reinforcing Stimulus		Total
	Right Light	Left Light	
Right-Right	119	114	233
Right-Left	99	56	155
Left-Right	40	44	84
Left-Left	85	47	132
Total	343	261	604

Note.—The reinforcing stimulus follows the first key press and precedes the second key press.

Anderson and Grant (1957), and that from a pilot study, to yield a total  $N$  of 604. In all cases,  $S$ s had three previous practice trials in which both lights or neither light could come on, but not either light singly. There is a significant preference for the right key on Trial 1, but not on Trial 2. The probability that the second press is an alternation response is .57, significantly above chance by a binomial test. It will be noted that the data could also be interpreted in terms of the "win-?, lose-stay" strategy of Goodnow and Pettigrew (1955). A somewhat different treatment of these data has been made by Sternberg (1959a) in terms of a model embodying a perseverative tendency.

### DISCUSSION

This discussion will first compare the present results with those of other experiments, and then consider the mathematical model approach to probabilistic learning situations of the kind investigated here.

The most relevant comparison data are those of Engler (1958). Four conditions in her Exp. II had acquisition values of  $\pi_{11}$  identical with those of the present Groups 10, 50, 70, and 90. The numerical values of the terminal acquisition levels are in fairly good agreement except that, for Group 10, the present asymptote of .04 is considerably lower than the .11 figure obtained by Engler. Her Group G, with  $\pi_{11} = .25$ , also terminates somewhat higher than might be expected by interpolation in Table 1. The source of this discrepancy is not clear. However, it is somewhat disturbing in view of the generally good agreement among different experiments on asymptotic values for the frequency variable.

It is worth mentioning that Groups 0 and 100, which received continuous reinforcement in acquisition, were little different from Group 50 in the transfer trials, but had significantly different transfer asymptotes than the other alternation and repetition groups. This result is not inconsistent with the com-

mon finding of greater resistance to extinction after partial reinforcement if the transfer condition is considered as analogous to extinction. It is possible, of course, that Groups 0 and 100 would have reached different transfer asymptotes had they been given more than 100 acquisition trials.

The finding that Group 0 learned faster than Group 100 is the reverse of the result of Goodnow and Pettigrew (1956). A plausible explanation lies in the difference in the reinforcing stimulus situation. In their "two-armed bandit" task, the fact that the other response would have been correct on trials when  $S$  received no chip was established by instruction, but no special signal to this effect was given during the experiment as was done in the procedure here.

The results from the alternation tuples do not seem to be entirely in agreement with corresponding results of Nicks (1959). In particular,  $R'(0010) - R'(1010)$  has the value of about .2 for Groups 30, 50, and 70, (Table 6). Nicks' Table 2, however, shows values of the same statistic of .02 and .04. The tabulation procedures were the same in both cases, and the reliabilities of the data are such as to make the discrepancy significant. Nicks reports these data only for his 67:33 frequency condition so that the source of the discrepancy may stem from this difference in the experimental situations. In any event, the present data show that Nicks' emphasis on runs as the only important stimulus tuples is not justified in general.

The final comparison with existing data is restricted to Group 50. For this condition, a statistic obtained by doubling the difference between the repetition response level and the chance level is the same as a sequential statistic computed by Estes and Straughan (1954). This quantity has here an asymptotic value of about .24 which agrees with the average value of .24 obtained from their three frequency conditions. In the report of Anderson and Grant (1957), however, these values (there denoted by  $a$ ) tended to be somewhat larger. In all three of these ex-

periments it was found that the  $\alpha$  values were considerably lower in the early trials.

The predictions made by the present model, as well as by the revised model of Burke and Estes (1957), disagree with the experimental results in a number of respects. In the first place, the acquisition asymptotes at the lower  $\pi_{11}$  values are considerably below the theoretical values. Second, the analyses of the sequential dependencies have exhibited several deviations: the negative recency effect, which is commonly reported, was again found here; the alternation tuple data showed that the third preceding light in the stimulus tuple, *RLR*, had a greater effect than the second preceding light in the stimulus tuple, *RL*, a result which is contrary to both models; in addition, the analysis of the response dependencies suggested that knowledge of the previous response yielded more information about the present response than would be expected theoretically, at least for models similar to those considered here. Finally, the fact that the final levels in the transfer condition (in which all groups received the same treatment) were different, and showed no sign of converging, is definitely contrary to the models.

Although these discrepancies have been obtained in the conditional probability situation, it is evident that they are in large part applicable to models for the frequency situation as well. In particular, the  $\pi_{11} = .5$  condition is common to both the frequency and the conditional probability situations. Thus, although the conditioning process assumed in current mathematical models for the two-choice task may well occur, it would appear that the models neglect much of the underlying processes. Furthermore, since the mean learning curve comprises an average of the sequential increments and decrements in response probability, the agreement between obtained and predicted "matching solution" behavior must be considered fortuitous.

A necessary condition for constructing an adequate model is brought out by the

sequential dependency analyses. The negative recency effect and the alternation tuple results imply specifically that it will be necessary to take into account the reinforcing stimulus on more than the one preceding trial which was considered in the two models discussed here. The situation could then be considered as discrimination learning, with memory traces of the last several events serving as discriminative stimuli in a system of S-R associations. It would probably be desirable to allow an analogous trace representation of preceding responses in the model, although the existing evidence is not unambiguous on the necessity for this.

The formulation of the preceding paragraph is consistent with the claim of Hake and Hyman (1953) that *Ss* perceive and respond to specific sequences of events preceding each trial, and is perhaps not opposed to the notion of "strategies" used by Goodnow and Pettigrew (1955). However, it is apparent that the elucidation of the effective discriminative stimuli will not be as simple as has been thought. The specification of these stimuli will probably rely heavily on sequential dependency data, but these, as has been seen, include a number of automatic effects not directly indicative of underlying psychological processes. Indeed, it would appear that the dissection of these data cannot be carried very far without an exact model. Estes' (1950) conceptualization of the stimulus, together with the treatment of probabilistic discrimination learning given by Burke and Estes (1957), affords a possible method of attack. Thus, for instance, shorter tuples or tuples of greater perceptual impact would correspond to larger subsets of stimulus elements or to subsets of stimulus elements with greater sampling probabilities. The stimulus element approach would also be well suited to quantifying the similarity of the total stimulus trace on successive trials. Sternberg's (1959b) path-dependent models with many-trial dependencies represent a second possible approach to the problem.

It was expected that the sequential

dependency analyses would give the most incisive information about the behavior since they most directly reflect the trial-to-trial increments and decrements in the variables underlying the overt response. Nevertheless, the portion of these data which was immediately useful was not overly large. As has been seen, the nonparametric interpretation of such data is difficult when the behavior is as complicated as is the case here. The sequential dependencies are most useful in testing or evaluating parameters of particular models since any quantitative model must predict their magnitude. A general mathematical treatment of sequential dependencies for linear learning models has been given by Anderson (1959). It can be shown, for example, that the original Estes and Straughan (1954) model implies that the entries within each of Tables 5 and 6 should be constant and equal to  $\theta(1 - \theta)^2$ , and  $\theta(1 - \theta)^3$ , respectively. Corresponding expressions for the two models discussed here were not worked out since they failed on other grounds. However, the cited expressions point up the usefulness of the dependency data. It is hoped that the present results will be of use in testing future models.

The behavior in the transfer condition raises a serious problem for any theoretical treatment of the two-choice probabilistic situation. It will be recalled that in the transfer condition, in which all Ss received a common treatment, the various groups showed no sign of converging to a common level of repetition responses even after hundreds of trials. The data also indicated that this transfer effect was not smoothly related to acquisition treatment. From a qualitative point of view, it is apparent that the acquisition treatment has induced some permanent or semipermanent change in the Ss. However, the results of the sequential dependency analyses have shown that at least some of the various stimulus tuples must be considered separately. Consequently, since the induced change could be associated with any or all of these tuples,

it is not possible to get a clear specification of its locus on the basis of the existing data.

The same problem holds for the mathematical model approach, but in an even more acute way since quantitative results are desired. The trace stimulus formulation would be feasible if the traces are short range and do not extend back more than, say, a half dozen trials. However, the transfer behavior is a long-range effect spanning several hundred trials. It is clearly impracticable to allow directly for such long term traces in any model. The same difficulty would apply to Sternberg's (1959b) path-dependent models.

It is possible, of course, that the long range effects arise from systematic changes (e.g., Restle, 1955) in the populations of stimulus elements which give rise to the short range traces. If this is correct, the suggested development would still be practicable although it would require the incorporation of the laws of such stimulus change. There is some evidence which may very gingerly be interpreted as suggesting that such laws of stimulus change may not be too complicated. The fact that the between-groups differences in the higher order dependencies were not more marked indicates that the changes in the stimulus element populations may possibly be largely confined to the stimulus 2-tuples. However, such a simple resolution of the problem of long range effects would still be insufficient to salvage the two models discussed here.

It seems not unreasonable to suggest that the most promising present line of attack on the two-choice situation lies in an extensive empirical investigation of the behavior under a variety of transfer conditions. This would include the study of repetition responses in situations in which the light frequencies are the independent variables. The present data, and those of Engler (1958) and of Goodnow and Pettigrew (1955) who also used a transfer paradigm, cover only a small part of the picture. Until more knowledge of transfer behavior is available, spot checking of specific mod-

els is liable to be little better than shooting in the dark.

In future work, it might well be more efficient to procure large quantities of data for a few *Ss* so that reliable individual values of the sequential dependencies can be obtained. It is apparent from the present results that a number of parameters will be needed to account for the behavior, whether or not this is done in a formal mathematical way. Averaging data over *Ss* in order to increase reliability is only too likely to simultaneously confound the descriptive statistics with individual differences (Anderson, 1959). This increases the difficulty of interpreting the statistics as well as any model parameters derived from them.

#### SUMMARY

Two experiments on two-choice probability learning were reported which varied the first-order conditional probability in the stimulus event sequences. This variable is the probability that an event occurs on one trial, given that it occurred the previous trial. The dependent variable was predictions of that same event which had occurred the previous trial.

Experiment I used a  $3 \times 2$  design with three trial rates and two values of conditional probability in the 200 acquisition trials. Acquisition was followed by transfer to completely random sequences. The conditional probability groups reached significantly different acquisition asymptotes, and stable, but significantly different, response rates in transfer.

In Exp. II, 11 groups were given acquisition training with conditional probabilities ranging from 0 to 1. Excepting the two extreme groups, 300 acquisition trials, 200 transfer trials, and 60 recovery trials were given. Acquisition asymptote was an orderly function of conditional probability. Stable but unequal response rates in transfer were obtained, as in Exp. I. Spontaneous recovery was also obtained.

Detailed analyses of sequential dependencies in the stimulus-response sequences found, in addition to the "gambler's fallacy," strong effects of strings of consecutive alternations in the stimulus sequences. Some evidence for a determining effect of previous responses was obtained.

Two models based on statistical learning theory predicted asymptotic acquisition response levels quite well for the higher values of conditional probability, but did poorly at the very low values. The models also failed to account for the nonconvergence in transfer, and for much of the sequential dependency data. It was concluded that agreement between obtained and predicted "matching solution" behavior is fortuitous.

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(Received May 4, 1959)