Natural Language Processing

Lecture 10—10/1/2013

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Today

- More HMMs
  - Urns and Balls
  - Forward-Backward

Hidden Markov Models

- States $Q = q_1, q_2 \ldots q_{N;}$
- Observations $O = o_1, o_2 \ldots o_{N;}$
  - Each observation is a symbol from a vocabulary V = $\{v_1, v_2, \ldots v_V\}$
- Transition probabilities
  - Transition probability matrix A = $\{a_{ij}\}$

$$a_{ij} = P(q_t = j \mid q_{t-1} = i) \quad 1 \le i, j \le N$$

- Observation likelihoods
  - Output probability matrix B=$\{b_i(k)\}$

$$b_i(k) = P(X_t = o_k \mid q_t = i)$$

- Special initial probability vector π

$$\pi_i = P(q_1 = i) \quad 1 \le i \le N$$

3 Problems

- Given this framework there are 3 problems that we can pose to an HMM
  - Given an observation sequence and a model, what is the probability of that sequence?
  - Given an observation sequence and a model, what is the most likely state sequence?
  - Given an observation sequence, infer the best model parameters for a partial model

Problem 1

- The probability of a sequence given a model...

Computing Likelihood: Given an HMM $\lambda = (A, B)$ and an observation sequence O, determine the likelihood $P(O|\lambda)$.

  - Used in model development... How do I know if some change I made to the model is making it better
  - And in classification tasks
    - Word spotting in ASR, language identification, speaker identification, author identification, etc.
      - Train one HMM model per class
      - Given an observation, pass it to each model and compute P(seq|model).

Problem 2

- Most probable state sequence given a model and an observation sequence

Decoding: Given as input an HMM $\lambda = (A, B)$ and a sequence of observations $O = o_1, o_2, ..., o_T$, find the most probable sequence of states $Q = q_1 q_2 q_3 \ldots q_T$.

  - Typically used in tagging problems, where the tags correspond to hidden states
    - As we'll see almost any problem can be cast as a sequence labeling problem
  - Viterbi solves problem 2

Problem 3

- Infer the best model parameters, given a skeletal model and an observation sequence...
  - That is, fill in the A and B tables with the right numbers...
    - The numbers that make the observation sequence most likely
  - Useful for getting an HMM without having to hire annotators...

Solutions

- Problem 1: Forward
- Problem 2: Viterbi
- Problem 3: EM
  - Or Forward-backward (or Baum-Welch)

The Viterbi Algorithm

function VITERBI(*observations* of len T,*state-graph* of len N) **returns** *best-path*

create a path probability matrix *viterbi[N+2,T]*
for each state s **from** 1 **to** N **do** ; initialization step
  $viterbi[s,1] \leftarrow a_{0,s} * b_s(o_1)$
  $backpointer[s,1] \leftarrow 0$
for each time step t **from** 2 **to** T **do** ; recursion step
  for each state s **from** 1 **to** N **do**
    $viterbi[s,t] \leftarrow \max_{s'=1}^{N} viterbi[s',t-1] * a_{s',s} * b_s(o_t)$
    $backpointer[s,t] \leftarrow \operatorname{argmax}_{s'=1}^{N} viterbi[s',t-1] * a_{s',s}$
$viterbi[q_F,T] \leftarrow \max_{s=1}^{N} viterbi[s,T] * a_{s,q_F}$; termination step
$backpointer[q_F,T] \leftarrow \operatorname{argmax}_{s=1}^{N} viterbi[s,T] * a_{s,q_F}$; termination step
return the backtrace path by following backpointers to states back in time from $backpointer[q_F,T]$

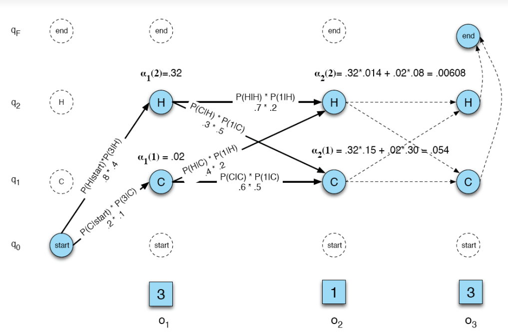
Forward

- Given an observation sequence return the probability of the sequence given the model...
  - Well in a normal Markov model, the states and the sequences are identical... So the probability of a sequence is the probability of the path sequence
  - But not in an HMM...

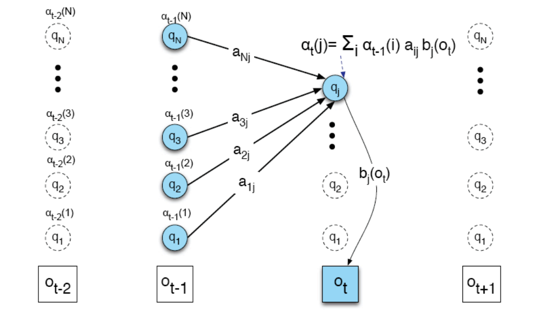
Forward

- Efficiently computes the probability of an observed sequence given a model
  - P(sequence|model)
- Nearly identical to Viterbi; replace the MAX with a SUM

Ice Cream Example



In General



Forward

function FORWARD(*observations* of len T, *state-graph* of len N) **returns** *forward-prob*

create a probability matrix *forward[N+2,T]*
for each state s **from** 1 **to** N **do** ; initialization step
  $forward[s,1] \leftarrow a_{0,s} * b_s(o_1)$
for each time step t **from** 2 **to** T **do** ; recursion step
  for each state s **from** 1 **to** N **do**

$$forward[s,t] \leftarrow \sum_{s'=1}^{N} forward[s',t-1] * a_{s',s} * b_s(o_t)$$

$$forward[q_F,T] \leftarrow \sum_{s=1}^{N} forward[s,T] * a_{s,q_F}$$; termination step

return $forward[q_F,T]$

Urn Example

- A genie has two urns filled with red and blue balls. The genie selects an urn and then draws a ball from it (and replaces it). The genie then selects either the same urn or the other one and then selects another ball…
  - The urns are hidden
  - The balls are observed

Urn

- Based on the results of a long series of draws...
  - Figure out the distribution of colors of balls in each urn
    - Observation probabilities (B table)
  - Figure out the genie's preferences in going from one urn to the next
    - Transition probabilities (A table)

Urns and Balls

- Pi: Urn 1: 0.9; Urn 2: 0.1
- A

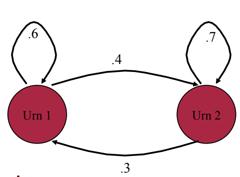
	Urn 1	Urn 2
Urn 1	0.6	0.4
Urn 2	0.3	0.7

- B

	Urn 1	Urn 2
Red	0.7	0.4
Blue	0.3	0.6

Urns and Balls

- Let's assume the input (observables) is Blue Blue Red (BBR)
- Since both urns contain red and blue balls any path of length 3 through this machine could produce this output



Urns and Balls

Blue Blue Red

1 1 1	(0.9*0.3)*(0.6*0.3)*(0.6*0.7)=0.0204
1 1 2	(0.9*0.3)*(0.6*0.3)*(0.4*0.4)=0.0077
1 2 1	(0.9*0.3)*(0.4*0.6)*(0.3*0.7)=0.0136
1 2 2	(0.9*0.3)*(0.4*0.6)*(0.7*0.4)=0.0181

2 1 1	(0.1*0.6)*(0.3*0.7)*(0.6*0.7)=0.0052
2 1 2	(0.1*0.6)*(0.3*0.7)*(0.4*0.4)=0.0020
2 2 1	(0.1*0.6)*(0.7*0.6)*(0.3*0.7)=0.0052
2 2 2	(0.1*0.6)*(0.7*0.6)*(0.7*0.4)=0.0070

Urns and Balls

Viterbi: Says 111 is the most likely state sequence

1 1 1	(0.9*0.3)*(0.6*0.3)*(0.6*0.7)=0.0204
1 1 2	(0.9*0.3)*(0.6*0.3)*(0.4*0.4)=0.0077
1 2 1	(0.9*0.3)*(0.4*0.6)*(0.3*0.7)=0.0136
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Urns and Balls

Forward: P(BBR| model) = .0792 Σ

1 1 1	(0.9*0.3)*(0.6*0.3)*(0.6*0.7)=0.0204
1 1 2	(0.9*0.3)*(0.6*0.3)*(0.4*0.4)=0.0077
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Urns and Balls

- EM
  - What if I told you I lied about the numbers in the model (Priors,A,B). I just made them up.
  - Can I get better numbers just from the input sequence?

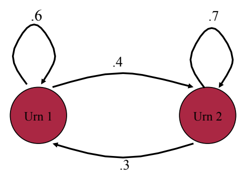
Urns and Balls

- Yup
  - Just count up and prorate the number of times a given transition is traversed while processing the observations inputs.
  - Then use that pro-rated count to re-estimate the transition probability for that transition

Urns and Balls

- But… we just saw that don't know the actual path the input took, its hidden!
  - So prorate the counts from all the possible paths based on the path probabilities the model gives you
    - Basically do what Forward does
- But you said the numbers were wrong
  - Doesn't matter; use the original numbers then replace the old ones with the new ones.

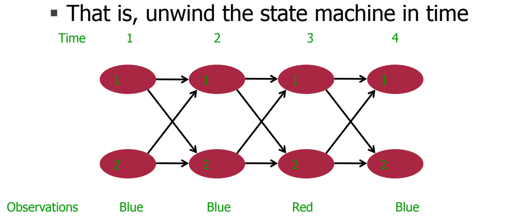
Urn Example



Let's re-estimate the Urn1->Urn2 transition and the Urn1->Urn1 transition (using Blue Blue Red as training data).

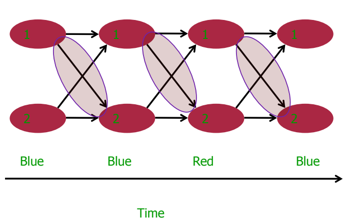
Another View

- We can view all those products as a lattice
  - That is, unwind the state machine in time



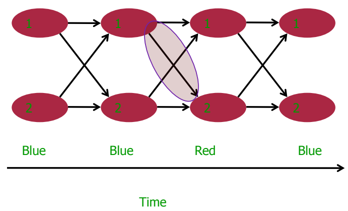
Another View

- Re-estimating the 1-> 2 transitions...



Another View

- Re-estimating the 1-> 2 transitions...



Urns and Balls

Blue Blue Red

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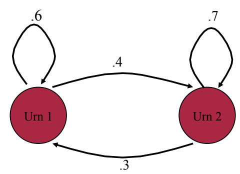
First, what exactly is this probability?

2 1 1	(0.1*0.6)*(0.3*0.7)*(0.6*0.7)=0.0052
2 1 2	(0.1*0.6)*(0.3*0.7)*(0.4*0.4)=0.0020
2 2 1	(0.1*0.6)*(0.7*0.6)*(0.3*0.7)=0.0052
2 2 2	(0.1*0.6)*(0.7*0.6)*(0.7*0.4)=0.0070

Urns and Balls

- So the probability of passing through 1->2 is the weighted sum of the paths taken through that transition given the observations
  - (.0077*1)+(.0136*1)+(.0181*1)+(.0020*1)
  - = .0414
- But, that's not the probability we want, it needs to be divided by the probability of leaving Urn 1 total.
- There's only one other way out of Urn 1 (going back to urn1)
  - So let's reestimate Urn1-> Urn1

Urn Example



Let's re-estimate the Urn1->Urn1 transition

Urns and Balls

Blue Blue Red

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Urns and Balls

- That's
  - (2*.0204)+(1*.0077)+(1*.0052) = .0537
- Again, not what we need but we're closer… we just need to normalize using those two numbers.

Urns and Balls

- The 1->2 transition probability is
  .0414/(.0414+.0537) = 0.435
- The 1->1 transition probability is
  .0537/(.0414+.0537) = 0.565
- So in re-estimation the 1->2 transition went up from .4 to .435 and the 1->1 transition went down from .6 to .565

EM Re-estimation

- Not done yet. No reason to think those values are right. They're just more right than they used to be.
  - So do it again, and again and....
  - Until convergence
  - Convergence does not guarantee a global optima, just a local one
- As with Problems 1 and 2, you wouldn't actually compute it this way. Enumerating all the paths is infeasible.

Break

- First quiz will be on October 6
  - Finite state automata/Morphology
  - N-Gram models
  - HMMs/POS tagging
  - Naïve Bayes/Logistic regression
  - Text classification/Sentiment analysis
- Office hours
  - Tuesday 11-12:30 and Friday 9:30 to 11

Forward-Backward

Learning: Given an observation sequence O and the set of possible states in the HMM, learn the HMM parameters A and B.

- **Baum-Welch** = **Forward-Backward Algorithm** (Baum 1972)
- Is a special case of the EM or Expectation-Maximization algorithm
- The algorithm will let us train the transition probabilities A= $\{a_{ij}\}$ and the emission probabilities B=$\{b_i(o_t)\}$ of the HMM

Intuition for re-estimation of a_{ij}

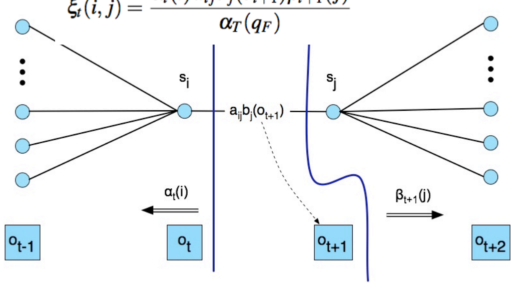
- We will estimate $\hat{a}_{ij}$ via this intuition:

$$\hat{a}_{ij} = \frac{\text{expected number of transitions from state } i \text{ to state } j}{\text{expected number of transitions from state } i}$$

- Numerator intuition:
  - Assume we had some estimate of probability that a given transition i➔j was taken at time t in a observation sequence.
  - If we knew this probability for *each* time *t*, we could sum over all *t* to get expected value for i➔j.

Intuition for re-estimation of a_{ij}

$$\xi_t(i,j) = \frac{\alpha_t(i)\, a_{ij} b_j(o_{t+1}) \beta_{t+1}(j)}{\alpha_T(q_F)}$$



Re-estimating a_{ij}

$$\hat{a}_{ij} = \frac{\text{expected number of transitions from state } i \text{ to state } j}{\text{expected number of transitions from state } i}$$

- The expected number of transitions from state *i* to state *j* is the sum over all *t* of ξ
- The total expected number of transitions out of state *i* is the sum over all transitions out of state *i*
- Final formula for reestimated a_{ij}

$$\hat{a}_{ij} = \frac{\sum_{t=1}^{T-1} \xi_t(i,j)}{\sum_{t=1}^{T-1} \sum_{j=1}^{N} \xi_t(i,j)}$$

The Forward-Backward Alg

function FORWARD-BACKWARD(*observations* of len *T*, *output vocabulary V*, *hidden state set Q*) **returns** *HMM=(A,B)*

initialize *A* and *B*
iterate until convergence

E-step

$$\gamma_t(j) = \frac{\alpha_t(j)\beta_t(j)}{\alpha_T(q_F)} \;\forall\, t \text{ and } j$$

$$\xi_t(i,j) = \frac{\alpha_t(i)\,a_{ij}b_j(o_{t+1})\beta_{t+1}(j)}{\alpha_T(q_F)} \;\forall\, t,\ i, \text{ and } j$$

M-step

$$\hat{a}_{ij} = \frac{\sum_{t=1}^{T-1} \xi_t(i,j)}{\sum_{t=1}^{T-1} \sum_{k=1}^{N} \xi_t(i,k)} \qquad \hat{b}_j(v_k) = \frac{\sum_{t=1 s.t. O_t=v_k}^{T} \gamma_t(j)}{\sum_{t=1}^{T} \gamma_t(j)}$$

return *A*, *B*

Summary: Forward-Backward Algorithm

1) Intialize Φ=(A,B)
2) Compute α, β, ξ using observations
3) Estimate new Φ' =(A,B)
4) Replace Φ with Φ'
5) If not converged go to 2

POS Example

- For practical applications we don't usually start with random numbers in the initial tables
- And we don't really run to convergence
- In POS
  - Use a dictionary to limit observation probabilities to allowable tag assignments
  - Use rough ordering of likely probabilities in initial assignments
  - Use a small amount of supervised data to get transition probabilities

Next time

- Read Chapter 7 from the draft 3Ed.
- Naïve Bayes and logistic regression for text classification and sentiment analysis