University of Colorado
Department of Computer Science

Computer Graphics – CSCI 4229
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Solutions to problem 1 of Problem Set 8

This is exactly like section 4.3.5 in the textbook, except that

$$\begin{aligned}\vec{u}_1 &= \vec{v}_3\\ \vec{u}_2 &= 2\vec{v}_1 + \vec{v}_3\\ \vec{u}_3 &= \vec{v}_3 - \vec{v}_2\end{aligned}$$

so the matrix M and its transpose M^T are:

$$M = \begin{bmatrix} 0 & 0 & 1 & 0\\ 2 & 0 & 1 & 0\\ 0 & -1 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \quad M^T = \begin{bmatrix} 0 & 2 & 0 & 0\\ 0 & 0 & -1 & 0\\ 1 & 1 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \tag{1}$$

The confusing part here is figuring out which to use: the matrix, its transpose, or its inverse-transpose.

The coordinates in the *old* basis are $a = [1, 2, 3, 0]^T$. What we want is the coordinates in the *new* basis, b. The equation for going from b to a is

$$a = M^T b$$

Going the other way, we need to use

$$b = (M^T)^{-1} a$$

In this case, we have a and we want b, so we need to use the second version of the equation, which means we need to compute the inverse of M^T:

$$(M^T)^{-1} = \begin{bmatrix} -1/2 & 1 & 1 & 0\\ 1/2 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \tag{2}$$

(Consult your linear algebra textbook if you don't remember how to find the inverse of a matrix. I did this one above by augmenting it with the identity matrix and doing elementary row operations to reduce the left-hand side to the identity – which took me two times to get right.)

Plugging all of this in:

$$b = \begin{bmatrix} -1/2 & 1 & 1 & 0\\ 1/2 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \begin{bmatrix} 1\\ 2\\ 3\\ 1\end{bmatrix} = \begin{bmatrix} -1/2 + 2 + 3\\ 1/2\\ -2\\ 1\end{bmatrix} \tag{3}$$