This is exactly like section 4.3.5 in the textbook, except that

\[
\begin{align*}
\vec{u}_1 &= \vec{v}_3 \\
\vec{u}_2 &= 2\vec{v}_1 + \vec{v}_3 \\
\vec{u}_3 &= \vec{v}_3 - \vec{v}_2
\end{align*}
\]

so the matrix \( M \) and its transpose \( M^T \) are:

\[
M = \begin{bmatrix}
0 & 0 & 1 & 0 \\
2 & 0 & 1 & 0 \\
0 & -1 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \quad M^T = \begin{bmatrix}
0 & 2 & 0 & 0 \\
0 & 0 & -1 & 0 \\
1 & 1 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (1)

The confusing part here is figuring out which to use: the matrix, its transpose, or its inverse-transpose. The coordinates in the old basis are \( a = [1, 2, 3, 0]^T \). What we want is the coordinates in the new basis, \( b \). The equation for going from \( b \) to \( a \) is

\[
a = M^T b
\]

Going the other way, we need to use

\[
b = (M^T)^{-1} a
\]

In this case, we have \( a \) and we want \( b \), so we need to use the second version of the equation, which means we need to compute the inverse of \( M^T \):

\[
(M^T)^{-1} = \begin{bmatrix}
-1/2 & 1 & 1 & 0 \\
1/2 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (2)

(Consult your linear algebra textbook if you don’t remember how to find the inverse of a matrix. I did this one above by augmenting it with the identity matrix and doing elementary row operations to reduce the left-hand side to the identity – which took me two times to get right.)

Plugging all of this in:

\[
b = \begin{bmatrix}
-1/2 & 1 & 1 & 0 \\
1/2 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
1 \\
2 \\
3 \\
1
\end{bmatrix} = \begin{bmatrix}
-1/2 + 2 + 3 \\
1/2 \\
-2 \\
1
\end{bmatrix}
\] (3)