Solutions to Problem Set 6

a. $\vec{a} \times \vec{b}$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & 3 & 0 \\ 1 & 4 & 0 \end{vmatrix} = \hat{x}\begin{vmatrix} 3 & 0 \\ 4 & 0 \end{vmatrix} - \hat{y}\begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} + \hat{z}\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = 5\hat{z}$$

b. Express $\vec{g} = (4, 7)$ as a linear combination of $\vec{h} = (3, 5)$ and $\vec{h}_\perp$

$\vec{g} = k\vec{h} + m\vec{h}_\perp$
Solve for k and m by multiplying by $\vec{h}$ and $\vec{h}_\perp$

$$\begin{aligned} \vec{g} \cdot \vec{h} &= k\vec{h} \cdot \vec{h} + m\vec{h}_\perp \cdot \vec{h} \\ k &= \frac{\vec{g} \cdot \vec{h}}{\vec{h} \cdot \vec{h}} \\ k &= \frac{47}{34} \end{aligned}$$

$$\begin{aligned} \vec{g} \cdot \vec{h}_\perp &= k\vec{h} \cdot \vec{h}_\perp + m\vec{h}_\perp \cdot \vec{h}_\perp \\ m &= \frac{\vec{g} \cdot \vec{h}_\perp}{\vec{h}_\perp \cdot \vec{h}_\perp} \\ m &= \frac{1}{34}\, if\ you\ used\ (5, -3) \\ m &= -\frac{1}{34}\, if\ you\ used\ (-5, 3) \end{aligned}$$

So,

$$\vec{g} = \frac{47}{34}\vec{h} + \frac{1}{34}\vec{h}_\perp \tag{1}$$

c. How far does point $P(6, 11)$ lie from the line through $Q(2, 5)$ and $R(4, -1)$?

$\vec{QR} = (2, -6)$

Distance from P to the line:

$$d = \frac{\left|\vec{QR}_\perp \cdot (P - Q)\right|}{\left|\vec{QR}_\perp\right|} = \frac{|(6,2) \cdot (4,6)|}{\sqrt{2^2 + (-6)^2}} = \frac{18}{\sqrt{10}} \approx 5.7$$

d. Normal vector to the plane that has points $P(1,1,1), Q(1,2,1), R(3,0,4)$

Find two vectors that are in the plane

$\vec{PQ} = (0,1,0)$ and $\vec{QR} = (2,-2,3)$

The normal vector is $\vec{PQ}$ x $\vec{QR}$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 1 & 0 \\ 2 & -2 & 3 \end{vmatrix} = \hat{x} \begin{vmatrix} 1 & 0 \\ -2 & 3 \end{vmatrix} - \hat{y} \begin{vmatrix} 0 & 0 \\ 2 & 3 \end{vmatrix} + \hat{z} \begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} = 3\hat{x} - 2\hat{z}$$

e. Parametric form of plane in d.

You need a point on the plane plus two vectors times two constants a and b that describe the plane.
$(1,1,1) + a(0,1,0) + b(2,-2,3)$

f. Parametric form of plane $2x - y + 3z = 8$

One possible set of three points in this plane is: $P(0,1,3), Q(5,2,0), R(2,1,2)$

Find two vectors

$\vec{PQ} = (5,1,-3)$ $\vec{PR} = (2,0,-1)$

Parametric form $= (0,1,3) + a(5,1,-3) + b(2,0,-1)$