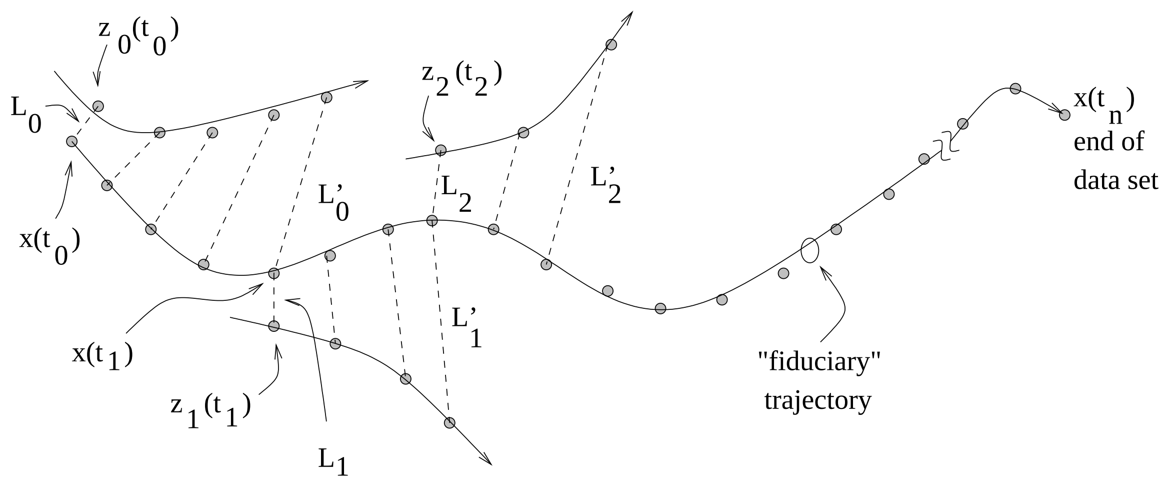
University of Colorado
Department of Computer Science

Chaotic Dynamics – CSCI 4446/5446

Wolf's algorithm for computing Lyapunov exponents from data:



Algorithm:

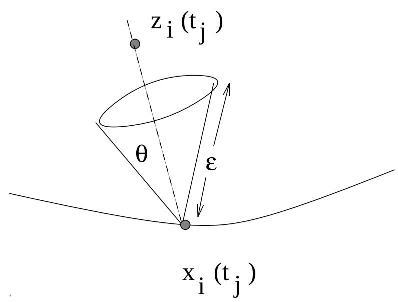
1. Embed the data set.
2. Pick a point $x(t_0)$ somewhere in the middle of the trajectory.
3. Find that point's nearest neighbor. Call that point $z_0(t_0)$.
4. Compute $||z_0(t_0) - x(t_0)|| = L_0$.
5. Follow the "difference trajectory" — the dashed line — forwards in time, computing $||z_0(t_i) - x(t_i)|| = L_0(i)$ and incrementing i, until $L_0(i) > \epsilon$. Call that value L'_0 and that time t_1.
6. Find $z_1(t_1)$, the "nearest neighbor" of $x(t_1)$, and loop to step 4. Repeat the procedure to the end of the fiduciary trajectory ($t = t_n$), keeping track of the L_i and L'_i.

Use this formula to compute λ_1, the biggest (positive) Lyapunov exponent:

$$\lambda_1 \approx \frac{1}{N\Delta t} \sum_{1}^{M-1} log_2 \frac{L'_i}{L_i}$$

...where M is the number of times you went through the loop above, and N is the number of timesteps in the fiduciary trajectory. $N\Delta t = t_n - t_0$.

Schematic of the directional "nearest neighbor" computation:



Start with $\theta = \pi/9$ and increase if necessary.

References:

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- Ott *et al.* *Coping with Chaos*, chapters 4 (an overview) and 7 (a bunch of reprinted journal papers). On library reserve.
- Parker & Chua, sections 3.3–3.4. On E-reserves.
- M. Sano, "Measurement of the Lyapunov Exponents from a Chaotic Series," *Physical Review Letters* **55**:1082-1085 (1983).
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- A. Wolf, "Quantifying chaos with Lyapunov exponents," in *Chaos*, Princeton University Press, 1986.