Conditional Probability Practice

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Dice

A die is rolled twice

what is the probability that the sum of the faces is greater than 7, given that

- the first outcome was 4?
- the first outcome was greater than 4?
- the first outcome was a 1?
- the first outcome was less than 5?

Dice

- $p(X_1 + X_2 > 7 \,|\, X_1 = 4) =$
- $p(X_1 + X_2 > 7 \,|\, X_1 > 4) =$
- $p(X_1 + X_2 > 7 \,|\, X_1 = 1) =$
- $p(X_1 + X_2 > 7 \,|\, X_1 < 5) =$

Dice

- $p(X_1 + X_2 > 7 \mid X_1 = 4) = \frac{p(X_1 + X_2 > 7 \wedge X_1 = 4)}{p(X_1 = 4)} =$
- $p(X_1 + X_2 > 7 \mid X_1 > 4) =$
- $p(X_1 + X_2 > 7 \mid X_1 = 1) =$
- $p(X_1 + X_2 > 7 \mid X_1 < 5) =$

Dice

- $p(X_1 + X_2 > 7 | X_1 = 4) = \frac{p(X_1+X_2>7 \wedge X_1=4)}{p(X_1=4)} = \frac{3/36}{1/6} = \frac{1}{2}$
- $p(X_1 + X_2 > 7 | X_1 > 4) =$
- $p(X_1 + X_2 > 7 | X_1 = 1) =$
- $p(X_1 + X_2 > 7 | X_1 < 5) =$

Dice

- $p(X_1 + X_2 > 7 \,|\, X_1 = 4) = \frac{p(X_1 + X_2 > 7 \wedge X_1 = 4)}{p(X_1 = 4)} = \frac{3/36}{1/6} = \frac{1}{2}$
- $p(X_1 + X_2 > 7 \,|\, X_1 > 4) = \frac{p(X_1 + X_2 > 7 \wedge X_1 > 4)}{p(X_1 > 4)} =$
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- $p(X_1 + X_2 > 7 \,|\, X_1 < 5) =$

Dice

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- $p(X_1 + X_2 > 7 | X_1 > 4) = \frac{p(X_1+X_2>7 \wedge X_1>4)}{p(X_1>4)} = \frac{9/36}{1/3} = \frac{27}{36} = \frac{3}{4}$
- $p(X_1 + X_2 > 7 | X_1 = 1) =$
- $p(X_1 + X_2 > 7 | X_1 < 5) =$

Dice

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- $p(X_1 + X_2 > 7 | X_1 = 1) = \frac{p(X_1+X_2>7 \wedge X_1=1)}{p(X_1=1)} =$
- $p(X_1 + X_2 > 7 | X_1 < 5) =$

Dice

- $p(X_1 + X_2 > 7 | X_1 = 4) = \frac{p(X_1+X_2>7 \wedge X_1=4)}{p(X_1=4)} = \frac{3/36}{1/6} = \frac{1}{2}$
- $p(X_1 + X_2 > 7 | X_1 > 4) = \frac{p(X_1+X_2>7 \wedge X_1>4)}{p(X_1>4)} = \frac{9/36}{1/3} = \frac{27}{36} = \frac{3}{4}$
- $p(X_1 + X_2 > 7 | X_1 = 1) = \frac{p(X_1+X_2>7 \wedge X_1=1)}{p(X_1=1)} = \frac{0}{1/6} = 0$
- $p(X_1 + X_2 > 7 | X_1 < 5) =$

Dice

- $p(X_1 + X_2 > 7 | X_1 = 4) = \frac{p(X_1+X_2>7 \wedge X_1=4)}{p(X_1=4)} = \frac{3/36}{1/6} = \frac{1}{2}$
- $p(X_1 + X_2 > 7 | X_1 > 4) = \frac{p(X_1+X_2>7 \wedge X_1>4)}{p(X_1>4)} = \frac{9/36}{1/3} = \frac{27}{36} = \frac{3}{4}$
- $p(X_1 + X_2 > 7 | X_1 = 1) = \frac{p(X_1+X_2>7 \wedge X_1=1)}{p(X_1=1)} = \frac{0}{1/6} = 0$
- $p(X_1 + X_2 > 7 | X_1 < 5) = \frac{p(X_1+X_2>7 \wedge X_1<5)}{p(X_1<5)} =$

Dice

- $p(X_1 + X_2 > 7 | X_1 = 4) = \frac{p(X_1+X_2>7 \wedge X_1=4)}{p(X_1=4)} = \frac{3/36}{1/6} = \frac{1}{2}$
- $p(X_1 + X_2 > 7 | X_1 > 4) = \frac{p(X_1+X_2>7 \wedge X_1>4)}{p(X_1>4)} = \frac{9/36}{1/3} = \frac{27}{36} = \frac{3}{4}$
- $p(X_1 + X_2 > 7 | X_1 = 1) = \frac{p(X_1+X_2>7 \wedge X_1=1)}{p(X_1=1)} = \frac{0}{1/6} = 0$
- $p(X_1 + X_2 > 7 | X_1 < 5) = \frac{p(X_1+X_2>7 \wedge X_1<5)}{p(X_1<5)} = \frac{6/36}{2/3} = \frac{18}{64} = \frac{1}{4}$

Children

What is the probability a family of two children has two boys

- given that it has at least one boy?
- given that the first child is a boy?

Children

- $P(X_1 = \top, X_2 = \top \,|\, X_1 = \top \vee X_2 = \top) =$
- $P(X_1 = \top, X_2 = \top \,|\, X_1 = \top) =$

Children

- $P(X_1 = \top, X_2 = \top \,|\, X_1 = \top \vee X_2 = \top) = \frac{P(X_1=\top, X_2=\top)}{P(X_1=\top \vee X_2=\top)} =$
- $P(X_1 = \top, X_2 = \top \,|\, X_1 = \top) =$

Children

- $P(X_1 = \top, X_2 = \top \,|\, X_1 = \top \vee X_2 = \top) = \frac{P(X_1 = \top, X_2 = \top)}{P(X_1 = \top \vee X_2 = \top)} = \frac{1/4}{3/4} = \frac{1}{3}$
- $P(X_1 = \top, X_2 = \top \,|\, X_1 = \top) =$

Children

- $P(X_1 = \top, X_2 = \top \,|\, X_1 = \top \vee X_2 = \top) = \frac{P(X_1 = \top, X_2 = \top)}{P(X_1 = \top \vee X_2 = \top)} = \frac{1/4}{3/4} = \frac{1}{3}$
- $P(X_1 = \top, X_2 = \top \,|\, X_1 = \top) = \frac{P(X_1 = \top, X_2 = \top)}{P(X_1 = \top)} =$

Children

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- $P(X_1 = \top, X_2 = \top \,|\, X_1 = \top) = \frac{P(X_1=\top, X_2=\top)}{P(X_1=\top)} = \frac{1/4}{1/2} = \frac{1}{2}$

Conditional Probabilities

One coin in a collection of 65 has two heads. The rest are fair. If a coin, chosen at random from the lot and then tossed, turns up heads 6 times in a row, what is the probability that it is the two-headed coin?

Conditional Probabilities

- Let C be the coin chose ($\top$ for fake)
- Let H be the number of heads out of six

$$P(C = \top \,|\, H = 6) = \tag{1}$$

Conditional Probabilities

- Let C be the coin chose ($\top$ for fake)
- Let H be the number of heads out of six

$$P(C=\top \mid H=6) = \frac{P(C=\top \wedge H=6)}{P(H=6)} = \tag{1}$$

Conditional Probabilities

- Let C be the coin chose ($\top$ for fake)
- Let H be the number of heads out of six

$$P(C = \top \,|\, H = 6) = \frac{P(C = \top \wedge H = 6)}{P(H = 6)} = \frac{1/65}{1/65 + \frac{64}{65} \cdot \frac{1}{2^6}} = \tag{1}$$

Conditional Probabilities

- Let C be the coin chose ($\top$ for fake)
- Let H be the number of heads out of six

$$P(C = \top \mid H = 6) = \frac{P(C = \top \wedge H = 6)}{P(H = 6)} = \frac{1/65}{1/65 + \frac{64}{65} \cdot \frac{1}{2^6}} = \frac{1/65}{2/65} = \frac{1}{2} \tag{1}$$

Bayes Rule

There's a test for Boogie Woogie Fever (BWF). The probability of geting a positive test result given that you have BWF is 0.8, and the probability of getting a positive result given that you do not have BWF is 0.01. The overall incidence of BWF is 0.01.

1. What is the marginal probability of getting a positive test result?
2. What is the probability of having BWF given that you got a positive test result?

Bayes Rule

Let D be the disease, T be the test

- $P(T = \top) =$
- $P(D = \top \,|\, T = \top) =$

Bayes Rule

Let D be the disease, T be the test

- $P(T = \top) = \sum_{x=\top,\bot} P(T = \top, D = x) =$
- $P(D = \top \,|\, T = \top) =$

Bayes Rule

Let D be the disease, T be the test

- $P(T=\top) = \sum_{x=\top,\perp} P(T=\top, D=x) = 0.01 \cdot .8 + .99 \cdot .01 =$
- $P(D=\top \,|\, T=\top) =$

Bayes Rule

Let D be the disease, T be the test

- $P(T=\top) = \sum_{x=\top,\perp} P(T=\top, D=x) = 0.01 \cdot .8 + .99 \cdot .01 = 0.02$
- $P(D=\top \,|\, T=\top) =$

Bayes Rule

Let D be the disease, T be the test

- $P(T = \top) = \sum_{x=\top,\perp} P(T = \top, D = x) = 0.01 \cdot .8 + .99 \cdot .01 = 0.02$
- $P(D = \top \,|\, T = \top) = \frac{P(T=\top \,|\, D=\top) P(D=\top)}{P(T=\top)} =$

Bayes Rule

Let D be the disease, T be the test

- $P(T=\top) = \sum_{x=\top,\perp} P(T=\top, D=x) = 0.01 \cdot .8 + .99 \cdot .01 = 0.02$
- $P(D=\top \,|\, T=\top) = \frac{P(T=\top \,|\, D=\top)P(D=\top)}{P(T=\top)} = \frac{0.8 \cdot 0.01}{0.02} =$

Bayes Rule

Let D be the disease, T be the test

- $P(T = \top) = \sum_{x=\top,\perp} P(T = \top, D = x) = 0.01 \cdot .8 + .99 \cdot .01 = 0.02$
- $P(D = \top \,|\, T = \top) = \frac{P(T=\top \,|\, D=\top) P(D=\top)}{P(T=\top)} = \frac{0.8 \cdot 0.01}{0.02} = 0.4$