\[ T(n) = \begin{cases} 
1 & n = 1 \\
 aT(n/b) + D(n) & n > 1, \text{ } n \text{ } \text{a power of } b
\end{cases} \]

**F Master Theorem.** For any nondecreasing function \( f(n) \) and any \( d \geq 0, \)

\[
T(n) = \begin{cases} 
\Theta(D(n)) & D(n) = \Theta(n^d f(n)) \quad h < d \\
O(D(n) \log n) & D(n) = \Theta(n^h f(n)) \\
\Theta(n^h) & D(n) = O(n^d) \quad h > d
\end{cases}
\]

3. the middle case is tight, i.e., \( T(n) = \Theta(D(n) \log n) \) for \( D(n) = \Theta(n^h f(n)) \),
if \( f(n) \) satisfies this “flatness condition”:

\( f(\sqrt{n}) = \Omega(f(n)) \)

\( f(n) = \log n \) satisfies (F), \( f(n) = n \) doesn’t

the set of \( f \)'s satisfying (F) is closed under product, powers, logs

\( e.g., \log^2 n, \sqrt{\log n}, \log \log n \) satisfy (F)

we can also relax (F), requiring it only for sufficiently large \( n \)

**F Master Theorem for Unequal Subproblems.**

Consider any recurrence

\[ T(n) = \sum_{i=1}^{k} T(a_i n) + D(n) \]

where \( 0 < a_i < 1, i = 1, ..., k \) and \( D(n) = n^d f(n) \) for a nondecreasing function \( f(n) \).

(change the arguments \( a_i n \) to \( a_i n + A_i \) if you wish).

Set \( s = \sum_{i=1}^{k} a_i^d. \)

\[
T(n) = \begin{cases} 
\Theta(D(n)) & s < 1 \\
O(D(n) \log n) & s = 1 \\
\Theta(n^h) & s > 1
\end{cases}
\]

where \( h \) satisfies \( \sum_{i=1}^{k} a_i^h = 1. \)

*The middle case is tight, \( T(n) = \Theta(D(n) \log n) \), for \( s = 1 \) and \( f \) satisfying (F).*