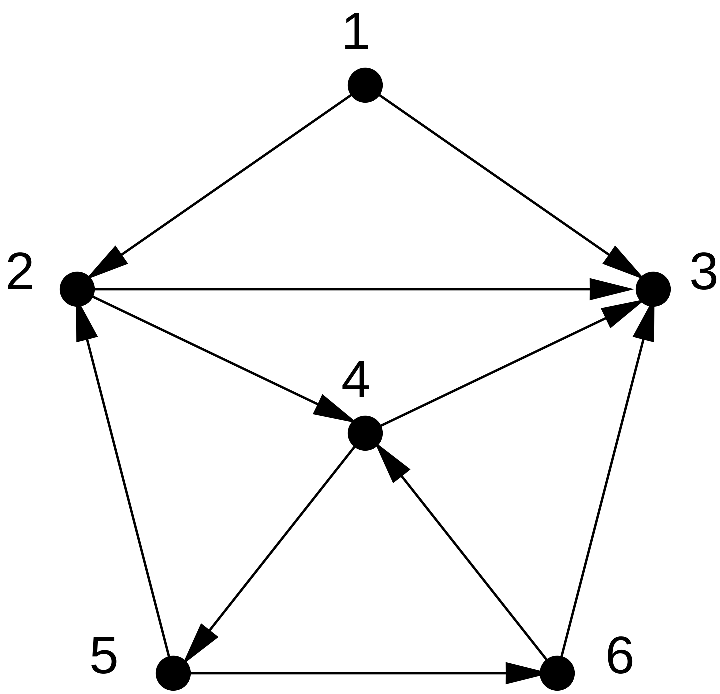
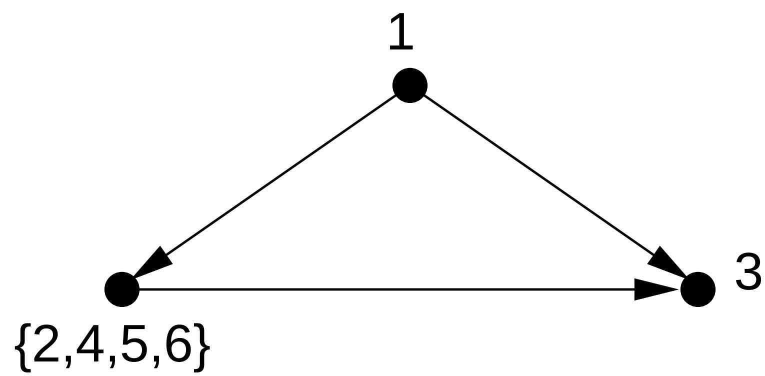
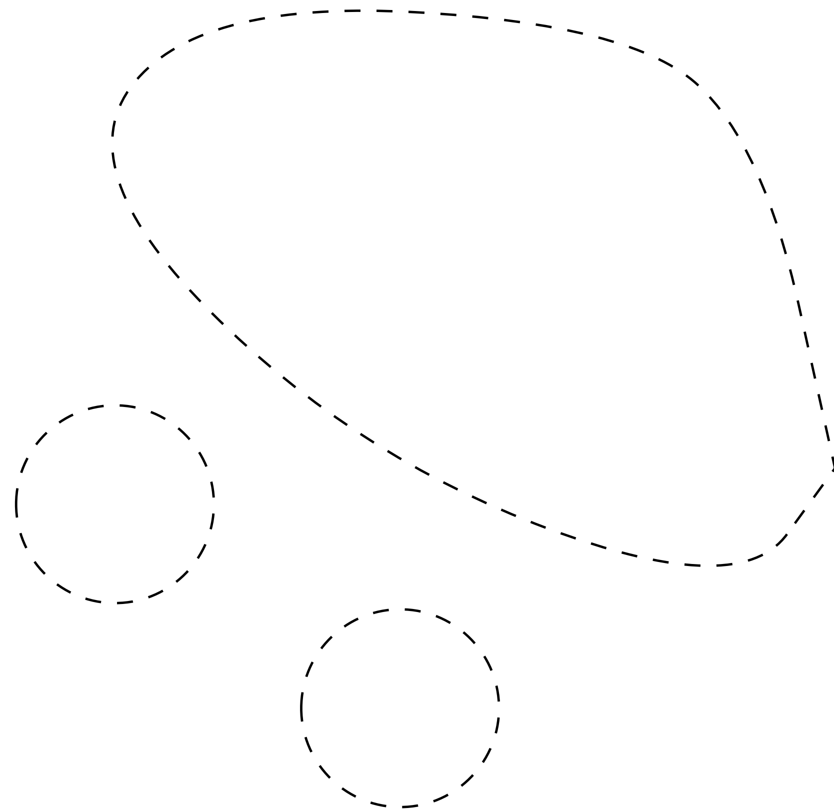
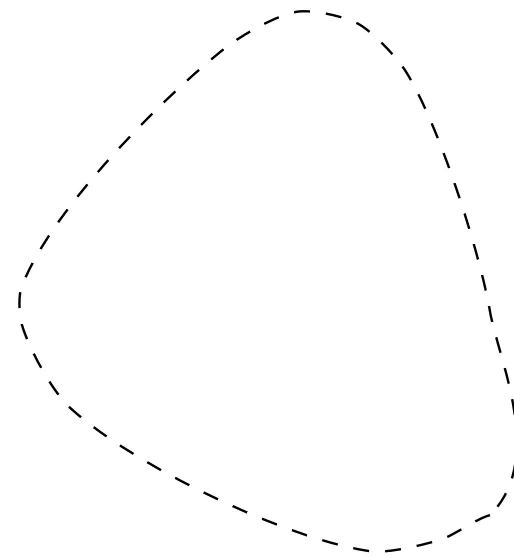
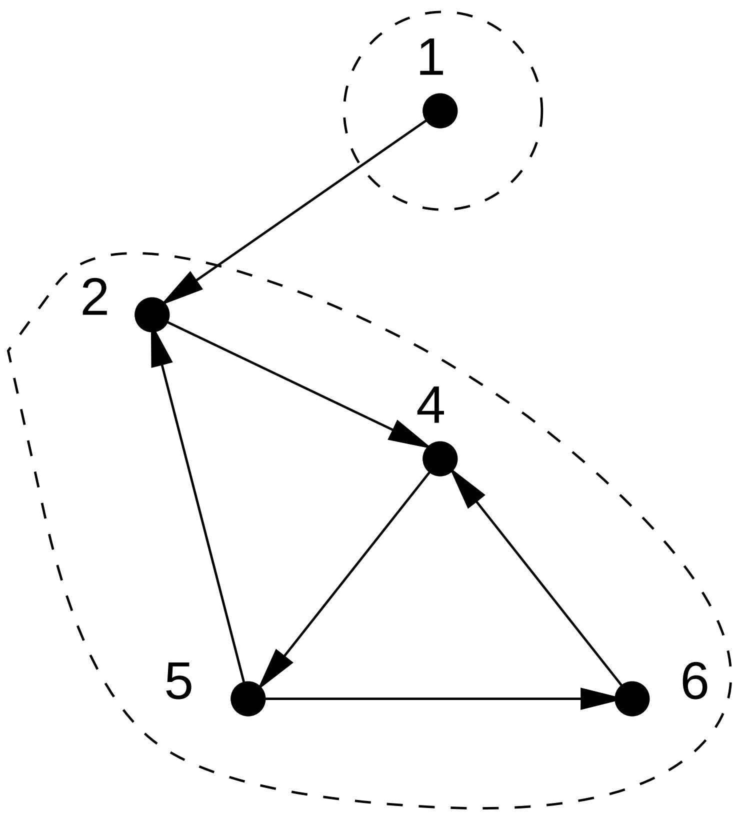




SC Graph





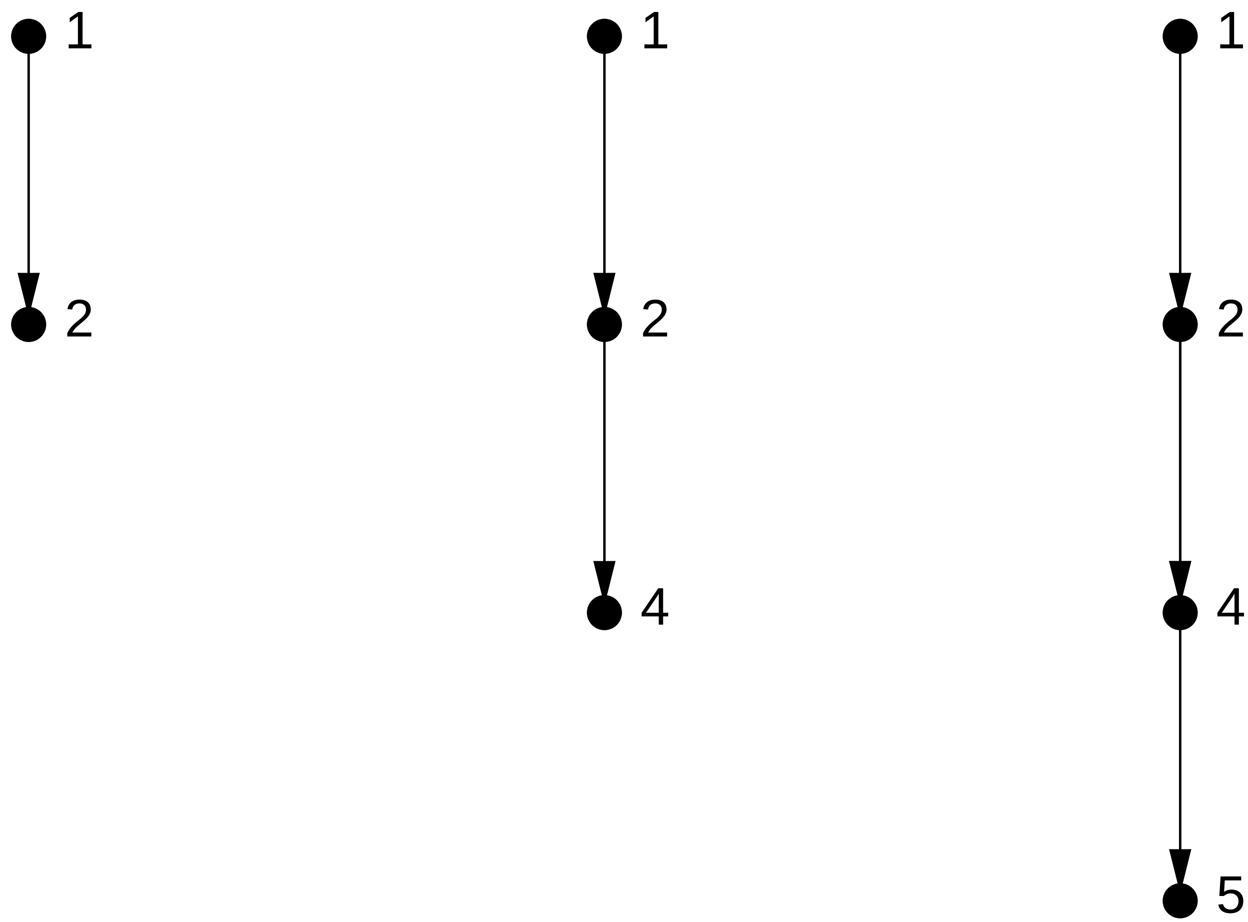

1
2
4
5
6

Strong Components

2 vertices are in the same SC $\Longleftrightarrow$ they can reach each other

Rules:

- the SC graph is formed by repeatedly contracting cycles
- a vertex with no leaving edge is a vertex of the SC graph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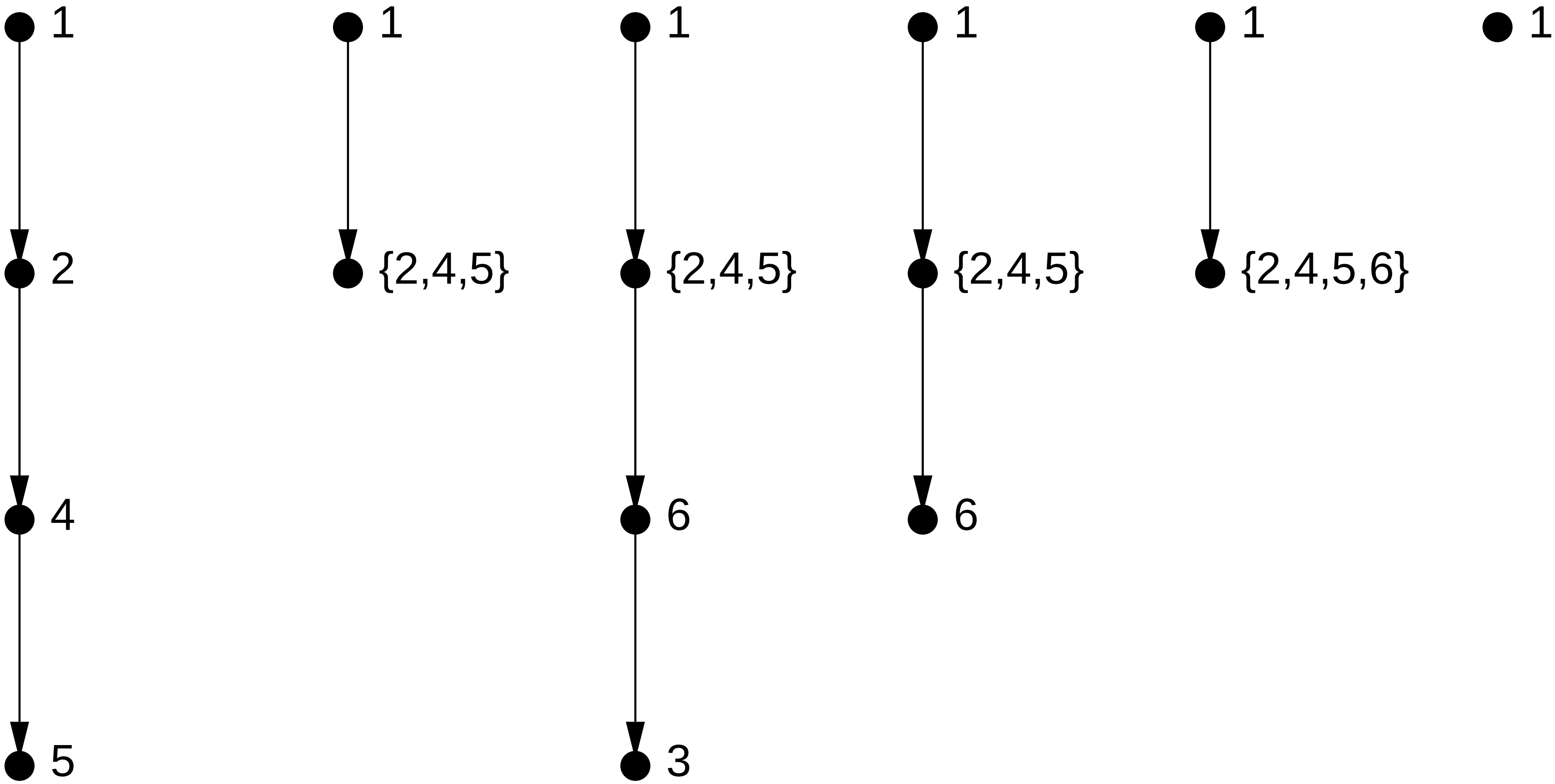
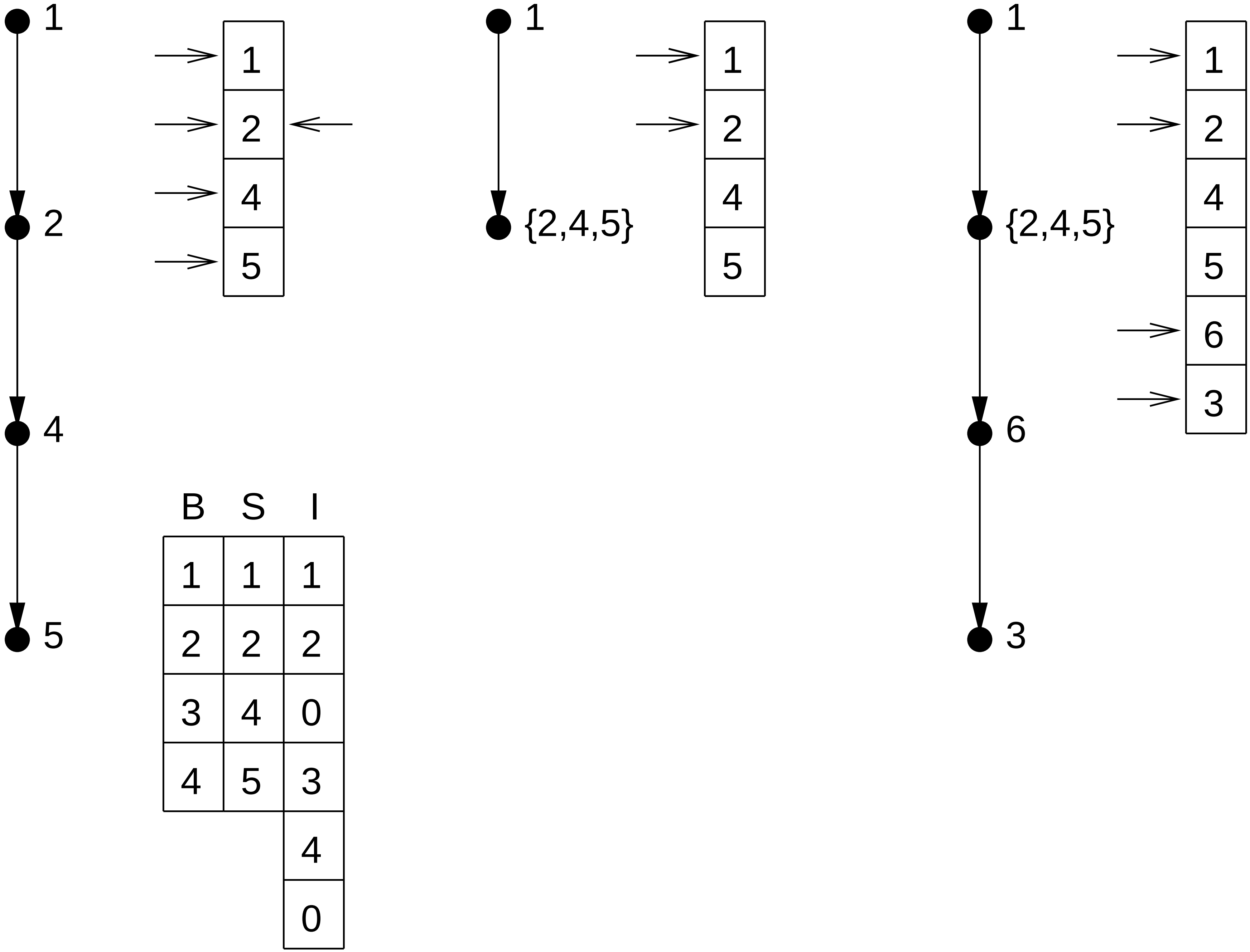
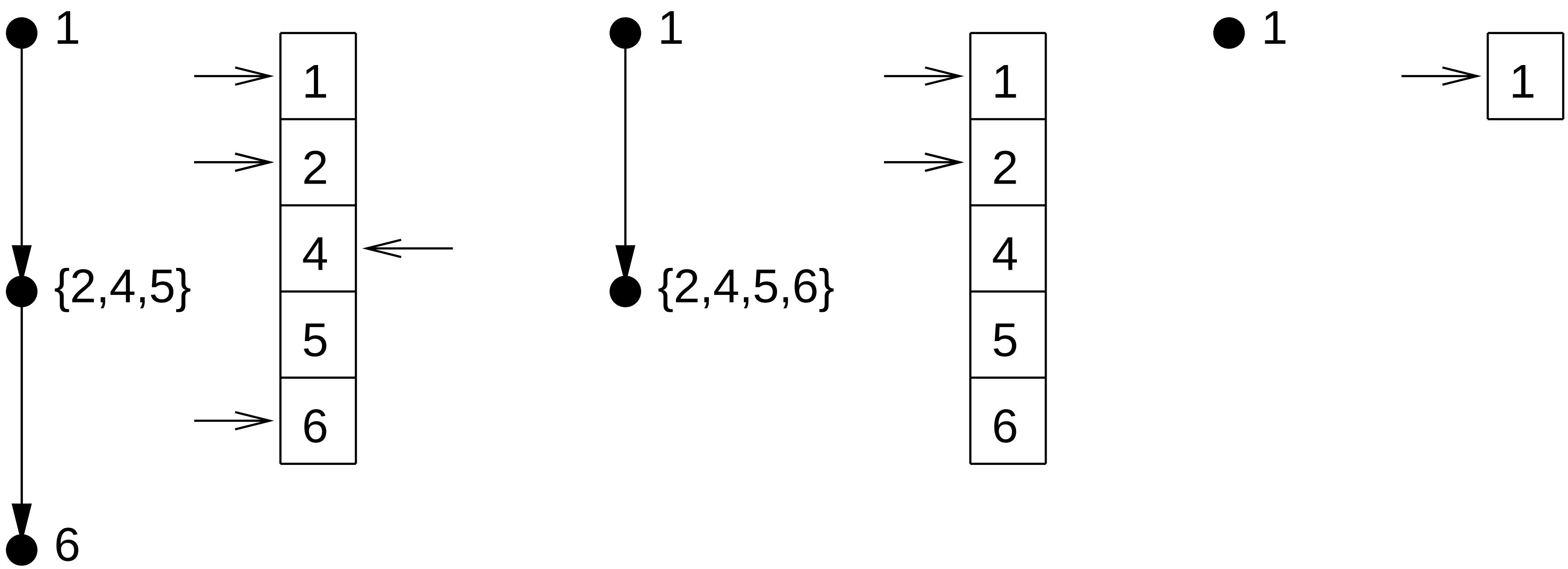
2
1
2
4
1
2
4
5


1
2
4
5
1
{2,4,5}
1
{2,4,5}
6
3
1
{2,4,5}
6
1
{2,4,5,6}
1


1
2
4
5
1 2 4 5
B S I
1 1 1
2 2 2
3 4 0
4 5 3
4
0
1
{2,4,5}
1 2 4 5
1
{2,4,5}
6
3
1 2 4 5 6 3


1
{2,4,5}
6
1
2
4
5
6
1
{2,4,5,6}
1
2
4
5
6
1
1

Strong Components Algorithm

procedure $\texttt{STRONG}(G)$

1. empty stacks S and B;
2. **for** $v \in V$ **do** $I[v] = 0$;
3. $c = n$;
4. **for** $v \in V$ **do if** $I[v] = 0$ **then** $\texttt{DFS}(v)$;

procedure $\texttt{DFS}(v)$

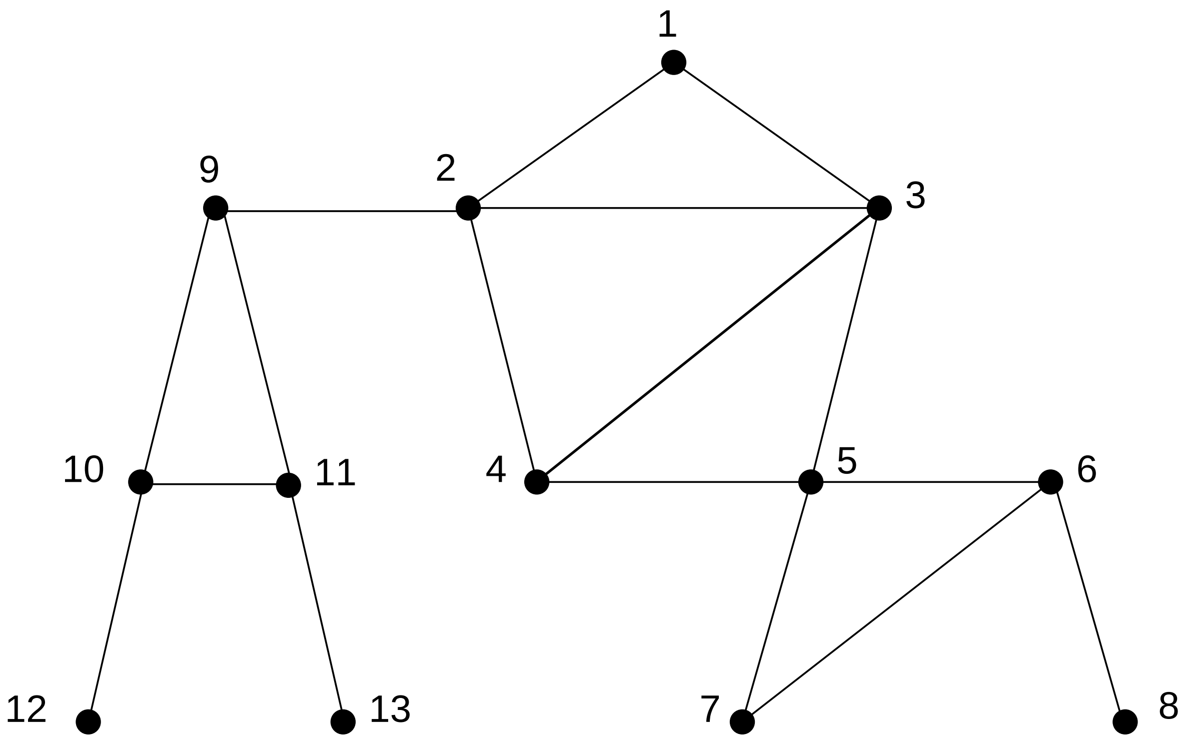
1. $\texttt{PUSH}(v, S)$; $I[v] = \texttt{TOP}(S)$; $\texttt{PUSH}(I[v], B)$;
2. **for** edges $(v, w) \in E$ **do**
3.   **if** $I[w] = 0$ **then** $\texttt{DFS}(w)$
4.   **else while** $B[\texttt{TOP}(B)] > I[w]$ **do** $\texttt{POP}(B)$;
5. **if** $B[\texttt{TOP}(B)] = I[v]$ **then** $\{$
6. $\texttt{POP}(B)$; increase c by 1;
7.   **while** $\texttt{TOP}(S) \geq I[v]$ **do** $\{\ w = \texttt{POP}(S);\ I[w] = c\ \}$ $\}$;

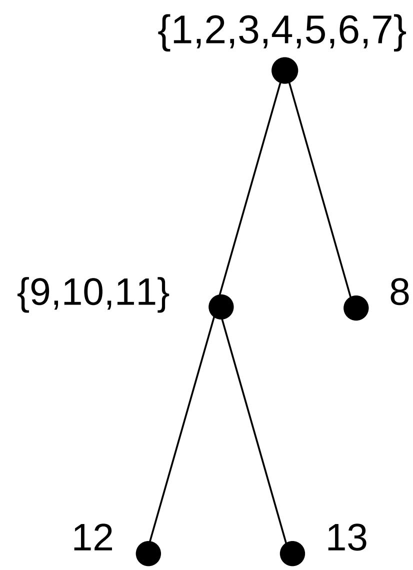
Biconnected Components

2 edges are in the same block $\Longleftrightarrow$ they are in a common cycle

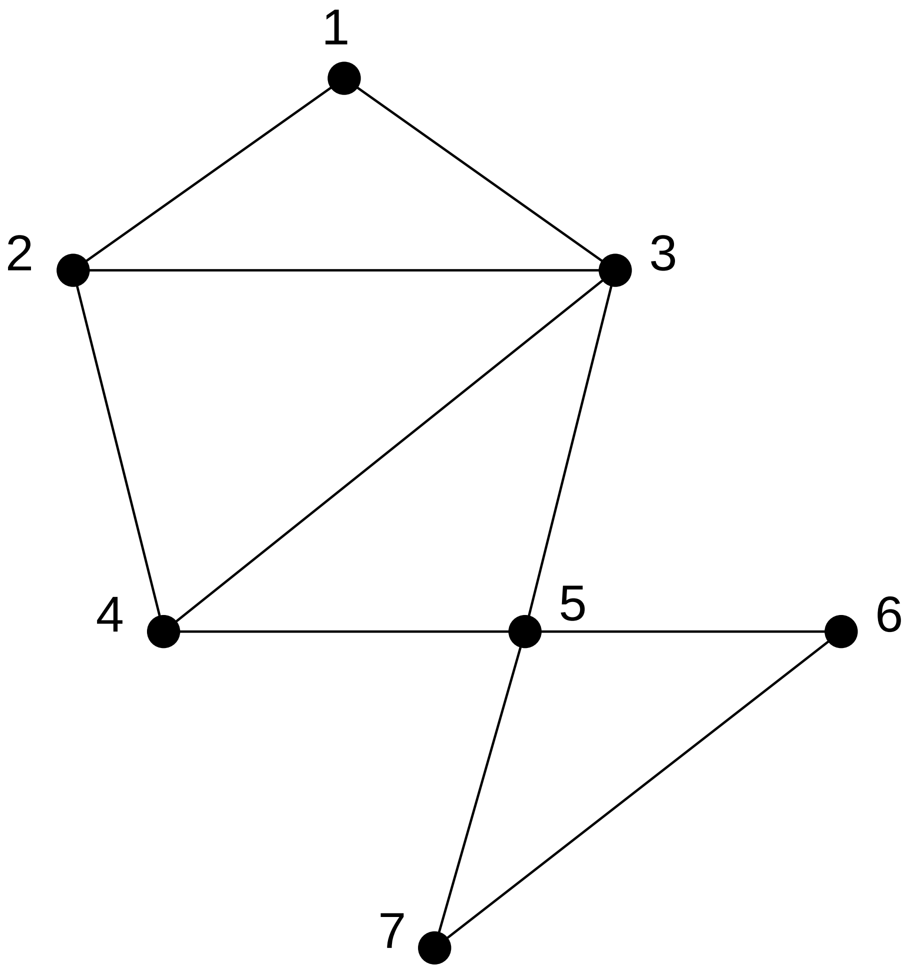
Rules:

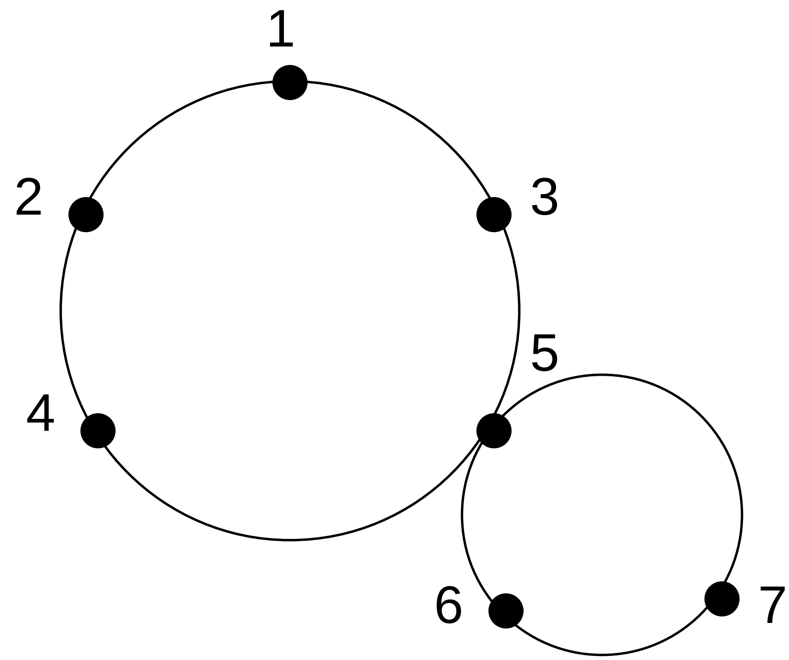
- the block hypergraph is formed by repeatedly merging cycles
- a hyperedge with 1 vertex on every incident edge is a block



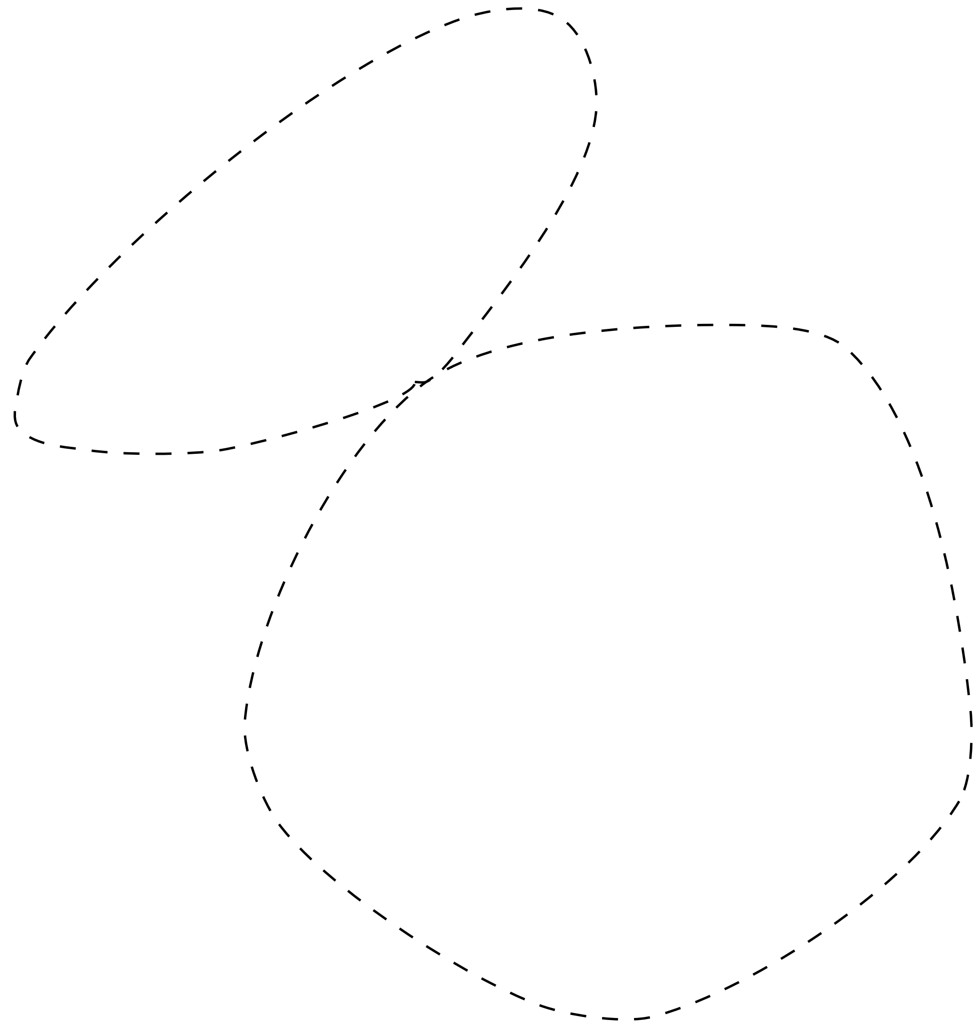


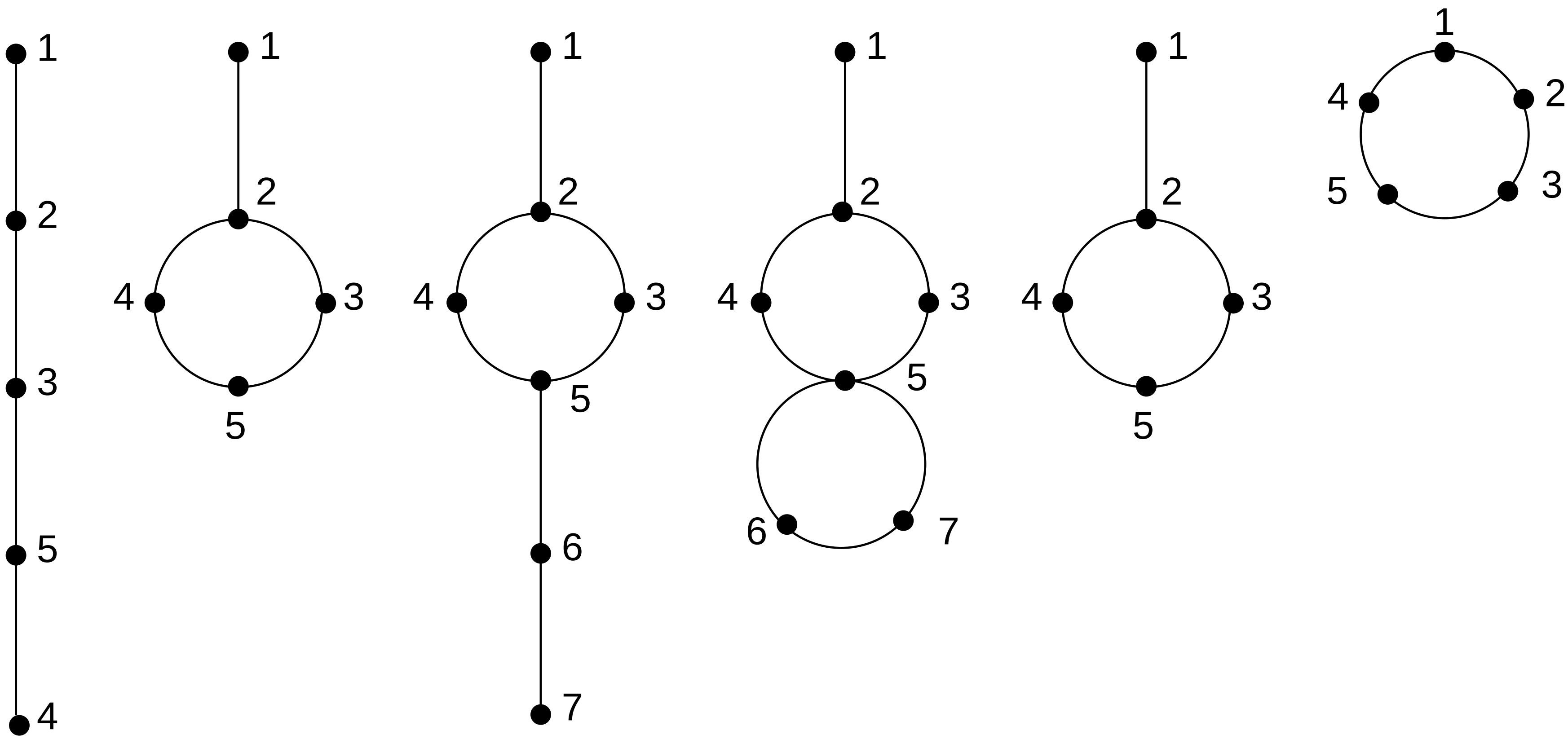
Bridge Tree

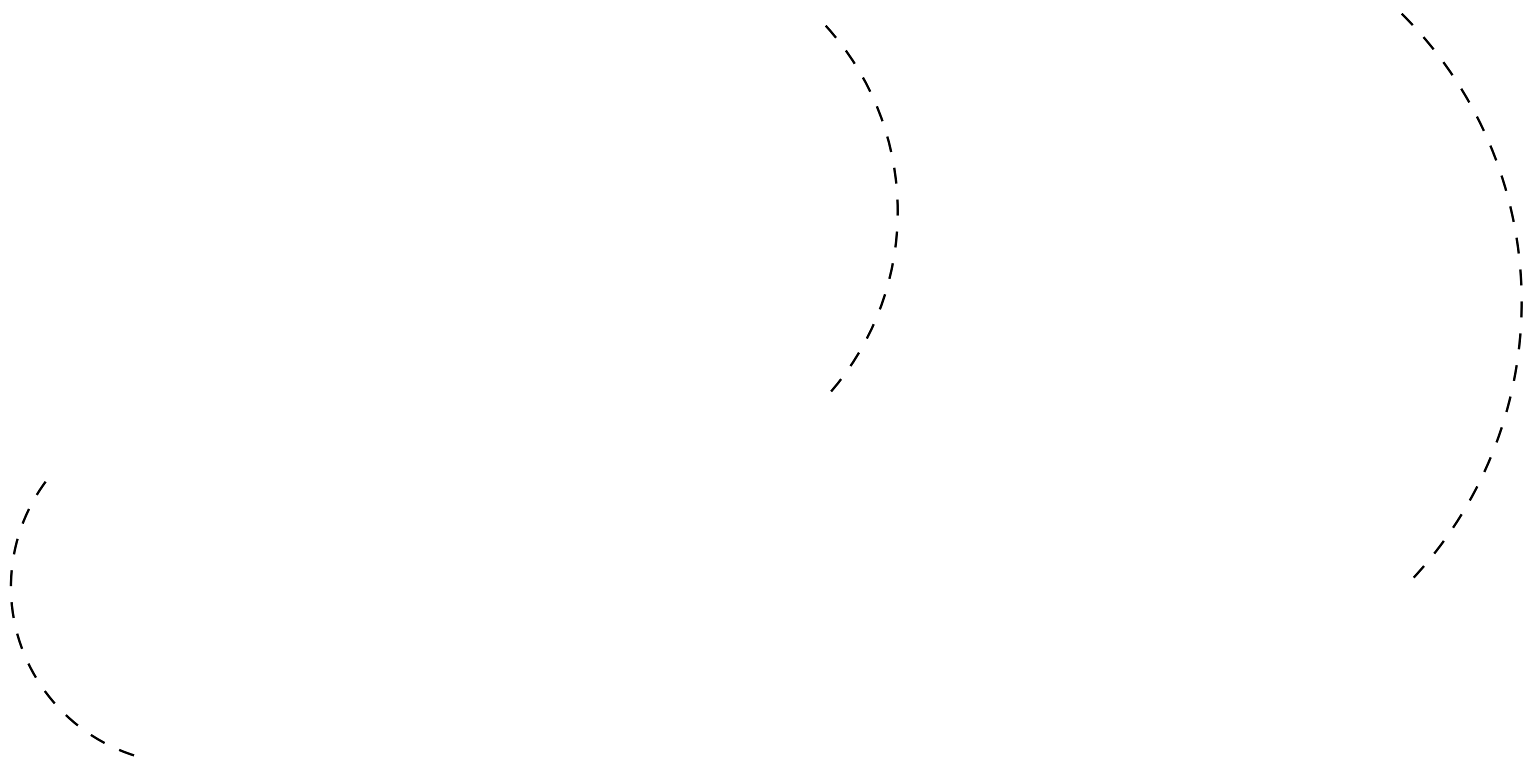


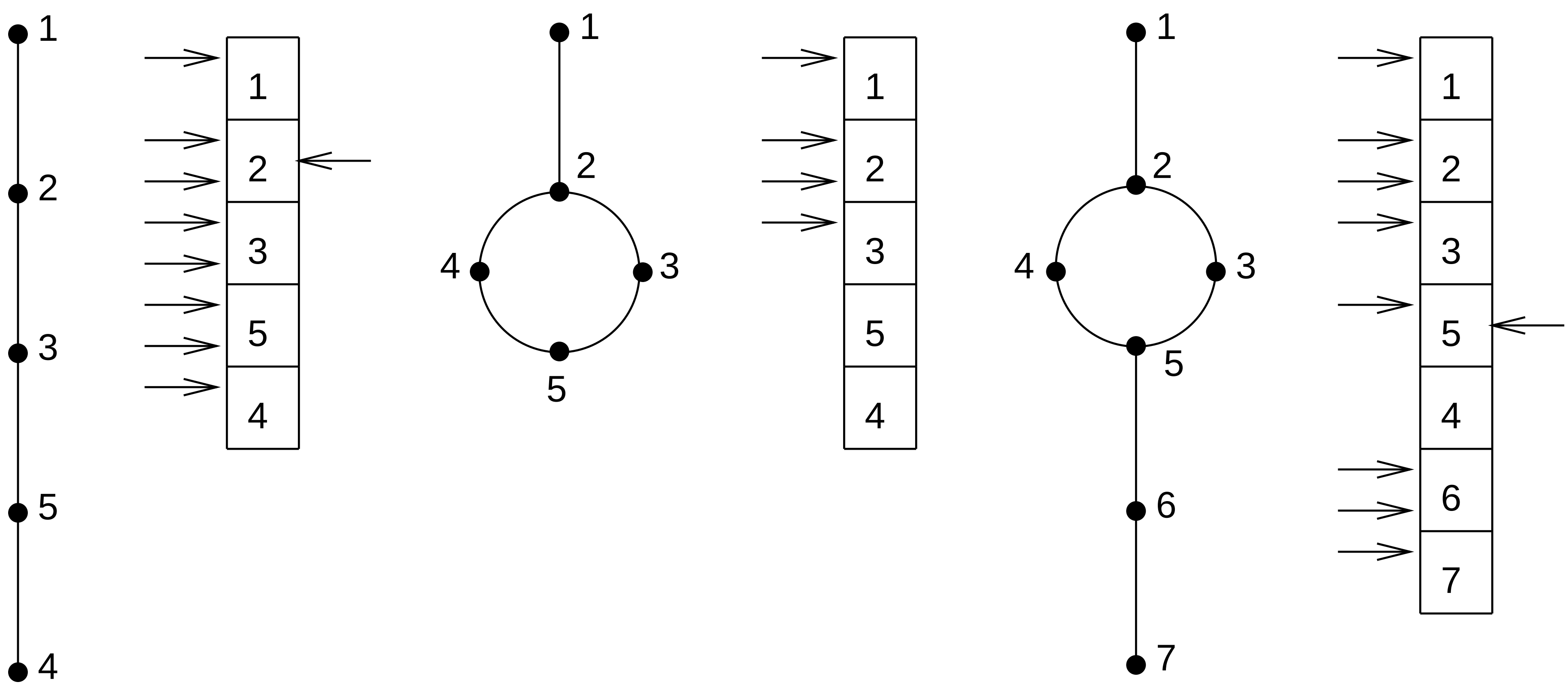


Block Hypergraph




1 2 3 5 4
1 2 4 3 5
1 2 4 3 5 6 7
1 2 4 3 5 6 7
1 2 4 3 5
1 2 4 5 3





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7
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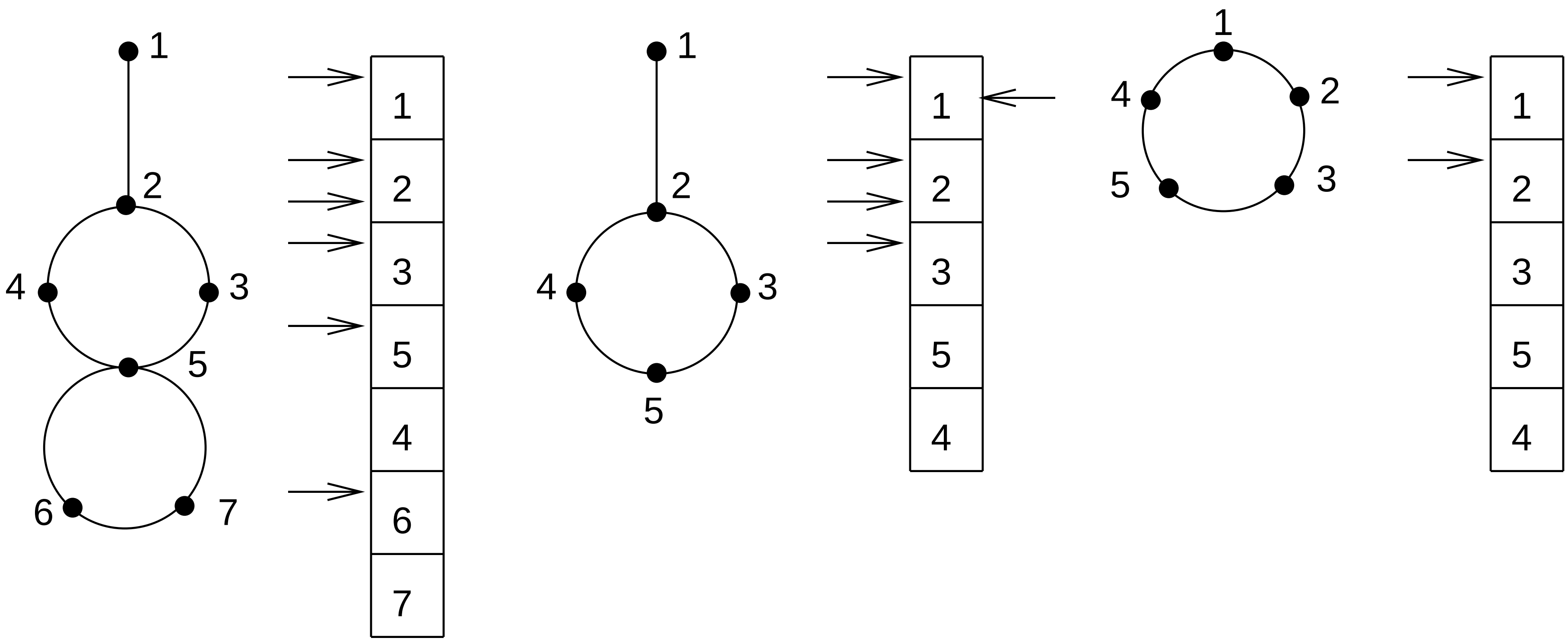
7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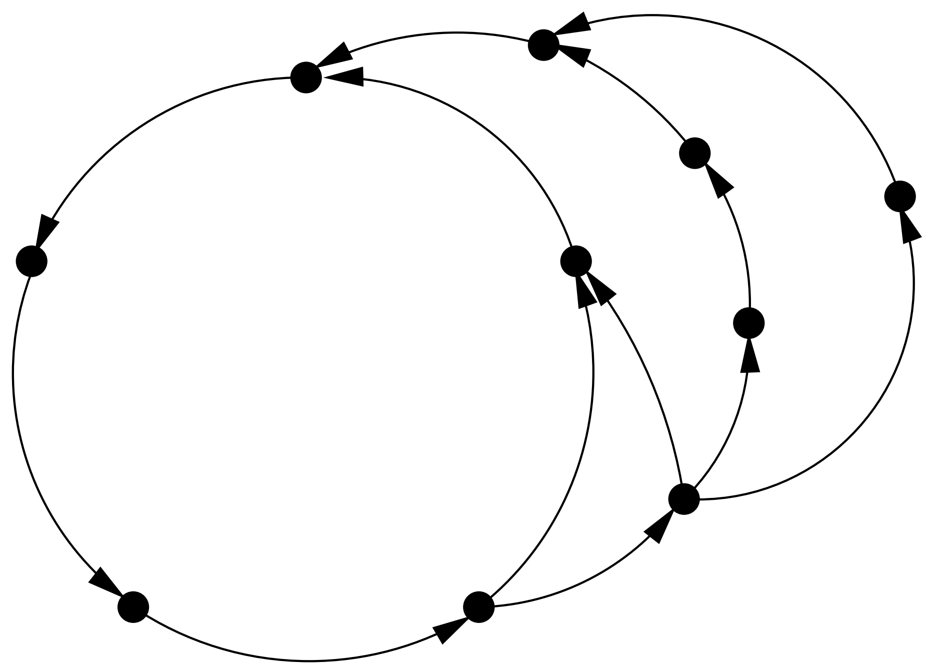
4

Other Applications (cont'd)

ear decomposition of strong, biconnected & bridgeless graphs



Ear decomposition of strongly connected graph