

Computational Geometries

Problem Set 5

Handed out: November 7, 2001

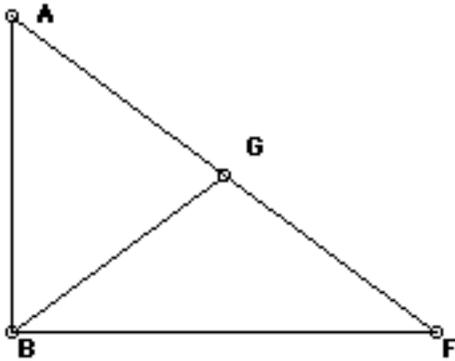
Due: Dec. 3, 2001

Problem 5.1 (30 pts)

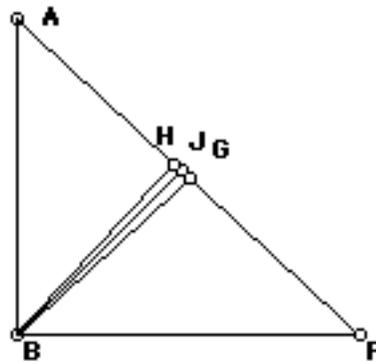
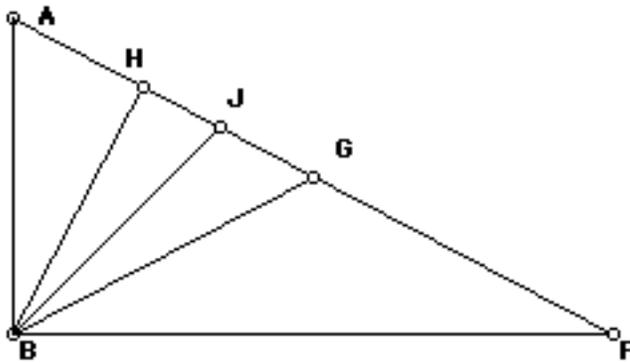
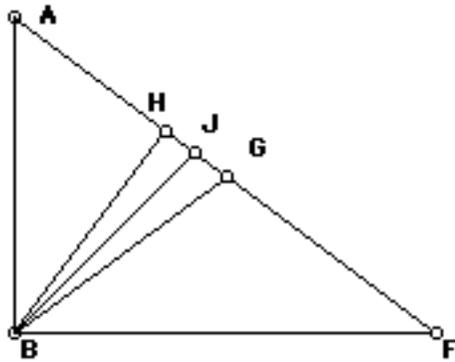
Based on a problem from Posamentier and Salkind

Use the Geometer's Sketchpad to produce diagrams for this problem and to help in solving it. (Your particular labels might be different, of course.)

- (a) Create a right triangle ABF and draw a line from the corner B (at the right angle) to the midpoint of the hypotenuse AF . Measure the length of BG and the length of AG . Form a conjecture from this measurement, and prove your conjecture.

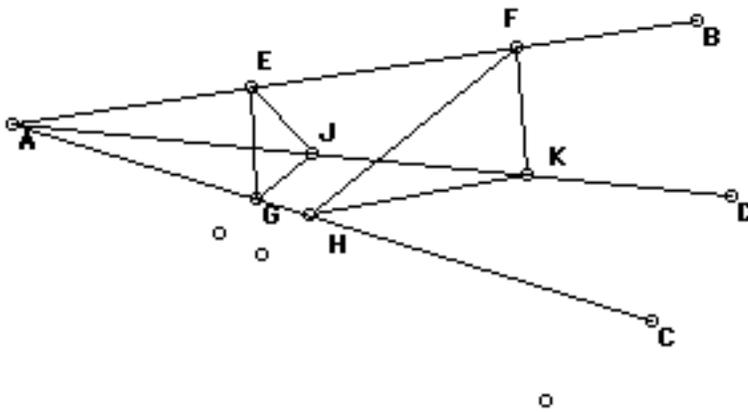


(b) Now add to this diagram the altitude BH from B to the hypotenuse; and the line BJ between B and the hypotenuse such that BJ is on the angle bisector of angle ABF . Measure the angles HBJ and JBG . Move either the vertex A or F in such a way that the triangle remains a right triangle, and again measure angles HBJ and JBG . Form a conjecture about this situation, and prove your conjecture.



Problem 5.2 (30 pts): *Desargues' Theorem.*

Start with an arbitrary point A. Construct three arbitrary line segments AB, AC, and AD as shown. On AB, choose two arbitrary points E and F; on AC, choose two arbitrary points G and H; on AD, choose two arbitrary points J and K. Draw the triangles EGJ and FHK. Now extend the corresponding lines of these triangles: find the intersection of the lines through EG and FH; through EJ and FK; through GJ and HK. (In the diagram below these three intersection points are shown, unlabelled.) Play with the original construction: move the points B, C, D; move the "interior" points E and F, G and H, J and K. State the (surprising!) conjecture that appears to jump out from this process.



Problem 5.3(40 pts)

In the attic of your home, you come upon a long-lost letter from a 17th-century ancestor, a well-known buccaneer who retired to a life of feigned respectability. Here is his inscription: "I have hidden a treasure chest of gold in the yard of my home. To find the treasure, do the following: Start at the oak tree. Walk toward the elm tree, measuring the distance D from the oak to the elm. Then turn left 90 degrees, walk the distance D once more, and note the point P. Then start once more at the oak tree. Walk toward the apple tree, measuring the distance D' from the oak to the apple. Turn right 90 degrees at the apple tree and walk the distance D' once more, and note point Q. The treasure is buried at the midpoint between P and Q."

This would seem like great news. You look out into the yard and see an elm and an apple tree toward either end of the yard. The only problem is, there is no oak tree! You remember at some point hearing that there was an oak tree, but it was cut down before the year 1900, and there is no trace of it remaining!

Using the Geometer's Sketchpad, investigate this situation, and see if you can

nonetheless find the treasure.