High-Level Hints

• Ex 2: Language is like IMP and like λ-calculus in ways
• Ex 3: Did "preservation" in class with big-step semantics
• Ex 4: Try an example derivation in ⊢ and "convert it" to ⊢₀
• Ex5: Think about using references: storing and reading

Exercise 4

• Consider the simply-typed lambda calculus with subtyping:

  \[ e ::= x \mid \lambda x. e \mid e_1 e_2 \]

  \[ \tau ::= \tau_1 \rightarrow \tau_2 \mid \ldots \]

• Typing rule for λ-expression:

  \[ \Gamma, x : \tau \vdash e : \tau' \]
  \[ \Gamma \vdash \lambda x. e : \tau \rightarrow \tau' \]
### Exercise 4

1. **Standard subsumption**
   
   \[ \Gamma \Downarrow e : \tau \quad \tau \Rightarrow \tau' \]
   
   \[ \Gamma \Downarrow e : \tau' \]

2. **Replaced with restricted variable-only subsumption**
   
   \[ \Gamma(x) = \tau \quad \tau \Rightarrow \tau' \]
   
   \[ \Gamma \Downarrow 0 x : \tau' \]

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### Exercise 4

- Prove "completeness of variable-only subsumption typing"

   \[ \text{If } T :: \vdash e : \tau, \text{ then } \vdash 0 e : \tau. \]

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### Exercise 4

If \( T :: \vdash e : \tau \), then \( \vdash 0 e : \tau \).

- Try proof by induction on \( T \).
- Case \( T :: \vdash \lambda x. e : \tau' \)

  Need generalization!

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### Exercise 4

**Corollary:** If \( T :: \vdash e : \tau \), then \( \vdash 0 e : \tau \).

- Try

  **Theorem:** If \( T :: \Gamma \vdash e : \tau \), then \( \Gamma \vdash 0 e : \tau \).

  - Resolves issue with \( \lambda \)-expression
  - If we can prove the theorem, then that (trivially) implies the corollary we want
  - Think about: The case where \( T \) ends in the subsumption rule.

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### Need generalization!

- Resolves issue with \( \lambda \)-expression
- If we can prove the theorem, then that (trivially) implies the corollary we want
- Think about: The case where \( T \) ends in the subsumption rule.