CSCI 3155, Recitation 1
Recursion and Induction

Warmup

1. Write a recursive function \( \log(b : \text{Int}, n : \text{Int}) : \text{Int} \) that computes \( \lfloor \log_b n \rfloor \).

2. Is our solution tail recursive? If not, can we write one that is tail recursive?

3. (a) What is the advantage of the tail recursive solution?
(b) What advantages can a non-tail recursive solution have?

4. Write a recursive function \( \text{sum}(n : \text{Int}) : \text{Int} \) that returns the sum of the integers from 1 to \( n \).

5. Prove by mathematical induction that your \( \text{sum}() \) function returns the sum values from 1 to \( n \) (hint: a closed form formula for the sum of the values from 1 to \( n \) is \( n \times (n + 1)/2 \)).

Recursion

1. Write a recursive function \( \text{isPrime}(n : \text{Int}) : \text{Boolean} \) that returns true if and only if \( n \) is a prime number (hint: use a helper function).

Induction

1. Given the height \( h \) of a stack of cannonballs, we can compute the number of cannonballs in the stack using the formula \( C(h) = h^2 + (h - 1)^2 + \ldots + 1^2 \).
(a) Write a recursive function \( \text{countCannonballs}(h : \text{Int}) : \text{Int} \) that computes the number of cannonballs in a stack given the height of the stack. You can assume that all stacks will have at least one cannonball.
(b) Prove by mathematical induction that your function computes the same result as the cannonball relation.
For this stack, \( h = 5 \), so \( C(5) = 5^2 + 4^2 + 3^2 + 2^2 + 1^2 = 55 \) cannonballs.